Distribution System State Estimation: Numerical Issues

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Outline

1. Introduction;
2. DSSE Methodologies;
3. Results;
Outline

1. Introduction;
2. DSSE Methodologies;
3. Results;
Introduction

• State Estimation

  – Key function in Supervisory Control and Planning
  – Widely used at the transmission level
  – Real-time steady-state of the system
  – Topology
  – New measurements technology (PMUs)
Introduction

• Smart Distribution Systems
  – New developments are required
  – Transition to an Active Nature

• Transmission and Distribution System State Estimator
  – The philosophy is different from each other
  – State Estimation Formulations
Introduction

• Transmission Level

  – Balanced Operation (Positive Sequence)
  – Meshed operation
  – Enough Redundancy to ensure Observability and Bad Data Processing
  – Most of measurements: Active and Reactive Power
  – Phasor Measurements: Several Publications;
Introduction

• Distribution Level
  – Three-Phase imbalances
  – Radial or weakly-meshed operation
  – Real-time measurements are limited
  – Observability is not achieved
  – Historical Load Data is used as pseudomeasurements
  – Most of measurements: Currents
  – Phasor Measurements: limited number of publications;
Outline

1. Introduction;
2. **DSSE Methodologies**;
3. Results;
DSSE Methodologies

- Nonlinear Model

- WLS method

- Solution based on *Normal Equations*

\[
[z] = [h(x)] + \varepsilon
\]

\[
J = \sum_{i=1}^{m} \left( \frac{z_i - h_i(x)}{\sigma_i} \right)^2
\]
DSSE Methodologies

- Methodologies differ from each other
- **Choice of state variables**
- Methodologies:

1. \[ x = \begin{bmatrix} |V| & \theta \end{bmatrix} \]  
   Nodal Voltages: Polar

2. \[ x = \begin{bmatrix} V_{\text{real}} & V_{\text{imag}} \end{bmatrix} \]  
   Nodal Voltages: Rectangular

3. \[ x = \begin{bmatrix} I_{km \text{ real}} & I_{km \text{ imag}} \end{bmatrix} \]  
   Branch Currents: Rectangular
DSSE Methodologies

Nodal Voltages based DSSE
DSSE Methodologies

• Nodal Voltage Polar Formulation
  – Three phase formulation
  – Current Magnitude measurement: squared to avoid numerical problems
  – PMU: Voltage measurement: Polar Coordinates

\[ h(x) = |I_{km}|^2 \]

\[ z = \begin{bmatrix} V_{PMU} & \theta_{PMU} \end{bmatrix} \]

– PMU: Current: Rectangular Coordinates

\[ z = \begin{bmatrix} I_{PMU}^{real} & I_{PMU}^{imag} \end{bmatrix} \]
DSSE Methodologies

• **Voltage Rectangular Formulation**

  – Three phase formulation

  – Current Magnitude measurement: squared to avoid numerical problems

  \[ h(x) = |I_{km}|^2 \]

  – PMU: Voltage measurement: Rectangular Coordinates

  \[
  z = \begin{bmatrix}
  V_{\text{real}}^{\text{PMU}} \\
  V_{\text{imag}}^{\text{PMU}}
  \end{bmatrix}
  \]

  – PMU: Current: Rectangular Coordinates

  \[
  z = \begin{bmatrix}
  I_{\text{real}}^{\text{PMU}} \\
  I_{\text{imag}}^{\text{PMU}}
  \end{bmatrix}
  \]
Branch Currents based DSSE
DSSE Methodologies

- Branch Current Formulation based DSSE
  - Three phase formulation
  - PMU: Voltage measurement: Rectangular Coordinates
    \[ z = \begin{bmatrix} V_{\text{real}}^{\text{PMU}} & V_{\text{imag}}^{\text{PMU}} \end{bmatrix} \]
  - PMU: Rectangular Coordinates
    \[ z = \begin{bmatrix} I_{\text{real}}^{\text{PMU}} & I_{\text{imag}}^{\text{PMU}} \end{bmatrix} \]
DSSE Methodologies

• Branch Current based DSSE
  – Power Flow and Injection Measurements: phase decoupling
    \[ h(x) = P_{km}^\phi = V_{\text{real}}^\phi I_{\text{real}}^\phi + V_{\text{imag}}^\phi I_{\text{imag}}^\phi \]
    \[ h(x) = Q_{km}^\phi = -V_{\text{real}}^\phi I_{\text{imag}}^\phi + V_{\text{imag}}^\phi I_{\text{real}}^\phi \]
  – Current Magnitude Measurements: phase decoupling
    \[ h(x) = |I_{km}^\phi| = \sqrt{(I_{km\text{ real}}^\phi)^2 + (I_{km\text{ imag}}^\phi)^2} \]
DSSE Methodologies

• Branch Current DSSE
  – Voltage Magnitude Measurements: **phase coupling**
  – The path from substation to the bus having the voltage meter is required.

Path: branches 1, 5 and 11.

\[ V = V_{sub} - Z_1 \cdot I_1 - Z_5 \cdot I_5 - Z_{11} \cdot I_{11} \]

\[ h(x) = |V| \]
DSSE Methodologies

• Branch Current DSSE

Implementation Issues

– In the pre-processing step the *Separation of the layers in the radial system* is used.
DSSE Methodologies

- Branch Current DSSE

Algorithm

- Step 1) Initialization:
  - Set the initial value of voltage at every node to 1pu
  - **Backward Step:** using the injected power at every node, the values of branch currents are obtained.
DSSE Methodologies

• Branch Current DSSE

Algorithm

– Step 2) Branch Current Estimation

- Calculate the state variable increments

\[
[G][\Delta x] = [\Delta y]
\]

- Update the value of state variables.

\[
x_{k+1} = x_k + \Delta x
\]
• Branch Current DSSE

Algorithm

– Step 3) Forward Step:

- Calculate the values of nodal voltages starting from the substation.
DSSE Methodologies

• Branch Current based DSSE

  Algorithm

  – Step 4) Convergence Analysis:
    - If the increments are smaller than a pre-specified tolerance: STOP
    - Otherwise:
      - If the number of iterations is smaller than the maximum number allowed, return to step 2.
      - Otherwise the convergence was not achieved.
Outline

1. Introduction;
2. DSSE Methodologies;
3. Results;
Results

- IEEE 34 Bus Test System

- Measurement Planning
  - Three Phase Power Flow Solution
  - Gaussian Error

- Simulations: MATLAB 2012b environment
Results

• Gaussian Errors

\[ z_{\text{meas}} = z_{\text{true}} + \text{rand.} \sigma \]

\[ \sigma = \frac{z_{\text{true}} \cdot \text{error}_\%}{300} \]

• Errors:
  • Power and current magnitude: 3%;
  • Voltage magnitude: 1%;
  • Phasor measurements: 0.5%;
  • Pseudomesurements (30%).
Subestation: Pair of Power Injection & Current Magnitude

Voltage Magnitude at buses 822, 848 & 890

PMUs: 816 & 834

Pseudomeasurements: Power injections at all buses
Results

Zero Injections:
All three phases have zero injections;
At least one phase have zero injection.
Results
Weights of zero Power injections
Measurements: Variation

Pseudomeasurements : 1E4
Initialization : Load Flow Solution/previous estimator solution
Results

Weights of Virtual Measurements: Variation

Pseudomeasurements : 1E4
Initialization : Flat Start
Results

Weights of Virtual Measurements: Variation

Pseudomeasurements: 1E4

Initialization: Power Flow Solution/Previous SE Solution

Gain Matrix Condition Number

- Polar
- Rectangular
- Current

Weights of Virtual Measurements:
- $10^4$
- $10^6$
- $10^8$
- $10^{10}$
- $10^{12}$
Results

Weights of Virtual Measurements: Variation

Pseudomeasurements: 1E4

Initialization: Flat Start

Gain Matrix Condition Number

Weights of Virtual Measurements

Polar
Rectangular
Current

NC
NC
NC
NC
Results

Weights of Virtual Measurements: Variation

Pseudomeasurements: 1E4

Computational Time (sec)

Initialization : Flat Start

<table>
<thead>
<tr>
<th>Weights of Virtual Measurements</th>
<th>$10^4$</th>
<th>$10^6$</th>
<th>$10^8$</th>
<th>$10^{10}$</th>
<th>$10^{12}$</th>
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<tbody>
<tr>
<td>Polar</td>
<td>7.317</td>
<td>9.292</td>
<td>10.080</td>
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<td>Rectangular</td>
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<tr>
<td>Current</td>
<td>0.806</td>
<td>0.795</td>
<td>0.722</td>
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<td>0.726</td>
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Initialization : Load Flow solution

<table>
<thead>
<tr>
<th>Average Time</th>
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<tr>
<td>Polar</td>
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<td>Rectangular</td>
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<tr>
<td>Current</td>
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</table>
Results
Three Phase MVA Base Variation

Polar

Rectangular

Current
Results: PMU Inclusion

Number of Iterations for various PMUs weights

<table>
<thead>
<tr>
<th>Weights of PMU Measurements</th>
<th>$10^4$</th>
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<tr>
<td>Polar</td>
<td>6</td>
<td>5</td>
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<tr>
<td>Rectangular</td>
<td>8</td>
<td>5</td>
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<tr>
<td>Current</td>
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<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Current</td>
<td>3</td>
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</tbody>
</table>
Outline

1. Introduction;
2. DSSE Methodologies;
3. Results;
Conclusions

General

• Improvements on Metering Infrastructure is required;
• Accuracy of component three-phase models;
Conclusions

General

• Measurement Weights Adjustment:
  – Extremely important;
  – Impact on convergence;
Conclusions

Implementation Complexity:

• Branch Current Formulation:
  – Derivatives: Simple Expressions;
  – Separation of the layers in the radial system;

• Nodal voltage formulations:
  – Derivatives: Complicated Expressions;
Conclusions

PMUs Inclusion (Complexity):

• Branch Current Formulation:
  – Voltage Phasor: Complicated (determination of layers);
  – Current Phasor: Simple;

• Nodal voltage formulations:
  – Voltage Phasor: Simple;
  – Current Phasor: complex derivatives
Conclusions

Computational Time:

• Branch Current based DSSE: faster than nodal voltage based methodologies;
Conclusions

Numerical Stability:

• **Branch Current Formulation**: condition number was found to be insensitive to weight variation;

• **Nodal voltage Formulations**: numerical stability depends on the measurement weights.
Conclusions

Three Phase Power Base Variation

• Branch Current based DSSE: the number of iterations was found to be insensitive;
• Nodal Voltages based formulation: the number of iterations increased according to the variation;
  – Polar formulation: the convergence was not achieved in some cases.
Conclusions

State Variables Initialization

• Branch Current based DSSE:
  – Power Flow Solution and Flat Start;
  – The number of iterations has slightly increased;

• Nodal Voltages based formulation:
  – Depends on the variable initialization;
  – Convergence may not be achieved.
Conclusions

Influence of PMUs

• Branch Current Formulation:
  – The convergence was found to be insensitive to the phasor measurement weight variation;

• Nodal Voltages formulation:
  – the number of iterations increased according to the weight variation;
  – Polar formulation: the convergence was not achieved in some cases.
Final Conclusions

• Other issues must be taken into account:
  – Radial and meshed operation;
  – Distribution Generation Inclusion;

• From the results obtained the branch current based DSSE is a promising methodology.
Acknowledgments
Questions?

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