Analytic Nonlinear Correction to the Impedance Boundary Condition

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Estimation of Stray Loss

• Leakage flux distribution and estimation of flux is a complex phenomenon

• The linear losses depend on the constant permeability and conductivity of the material. For magnetic steel materials, the B-H curve is non-linear and permeability varies with the field

• The surface impedance boundary condition is based on a loss formula for linear magnetic materials with high permeability and conductivity such that the losses are assumed to be confined to a thin region near the surface
Linear Losses

• The linear loss formula is developed for an infinitely thick slab of material. The fields are assumed to vary in the direction into the slab.

• Assuming the coordinate system shown in Fig., the fields vary only in the z-direction into the slab. The surface H-field is in the x-y plane.
Non-Linear Losses

• The B-H curve of structural steel is given in the figure. For comparison a linear B-H curve for a material with a relative permeability of 200 is also shown.

• The infinitely thick sheet was replaced by thickness ~ 5 to 10 times skin depth.
Non-Linear Losses

• The differential equation governing the magnetic field diffusion into the plate, is a function of the distance into plate and time.

• Given a sinusoidal surface, the field at each point in the material can be calculated from discretized differential equation.

• At each time step, the non-linear B-H curve is used to proceed to the next time step.

• The calculated H-field in the material, after convergence is achieved, is used to calculate the eddy currents and non-linear losses.
Finite Element Implementation

• Losses for the low carbon steel 1010 with the given B-H curve were calculated for a number of peak surface sinusoidal H fields at 60 Hz

• The ratio of the nonlinear to linear losses versus the peak surface H field plotted as shown in the figure.

• The best fit - a square root function - is not supported by FE calculator. The next best fit involved a linear function of the H-field and this gives an acceptable fit.

• This fit to the ratio of non-linear to linear losses is in the form of an analytic formula that is the correction factor used in the finite element program.
Ratio of nonlinear to linear losses
Comparison with Tested Losses

- The impedance boundary stray losses with the nonlinear correction generally agree with the tested stray losses than assuming a linear B-H curve.
- The ratio of nonlinear losses to linear losses is ~ 1.6

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| Ave | 0.357 | - 12.023 |
| StdDev | 10.603 | 12.976 |
Impedance boundary surface losses on 2 tank walls
(The key is in W/m²)
Conclusion

• The measurement of stray losses in a transformer is an indirect process. The winding $I^2R +$ eddy losses must be subtracted from the measured losses to get the stray losses. This can result in significant errors.

• The correction described is not only dependent on the B-H curve of the steel used but also on the linear permeability of the material used. It depends on surface H-field distribution and is not simply a single number which multiplies the total surface loss.

• It can be also applied to any other material, if magnetic and electrical characteristics are known.