Adjustable Robust Optimization for Contingency-Constrained Transmission Expansion Planning

José M. Arroyo*
Universidad de Castilla – La Mancha

*In collaboration with Alexandre Street (Pontifical Catholic University of Rio de Janeiro, Brazil) and Alexandre Moreira (Imperial College, London, UK)
Contents

• Introduction

• Adjustable robust optimization

• Solution methodology

• Numerical results

• Conclusions
Security-constrained transmission expansion planning

- Optimal location and sizing of candidate assets over a single stage ⇒ Static model

- Subject to:
  - Investment constraints
  - Operational constraints
  - Deterministic security criteria (industry practice) ⇒ n – 1, n – 2, ..., n – K
Conventional approach
Contingency-constrained model

Minimize $c(x)$

subject to:

$g(x) \leq 0$

$g_k(x, x_k) \leq 0 \quad \forall k \in C$
Contingency-constrained model

- Contingency \( k \Rightarrow \) Combination of available assets
- Asset availability \( \Rightarrow \) Binary parameters \( A_i^k \) (generator outages) and \( A_l^k \) (line outages)
  - 1 (available) / 0 (unavailable)
  - \( C \) determined by the security criterion:
    \[
    f\left(\{A_i^k\}_{i \in I}, \{A_l^k\}_{l \in L}\right) \geq 0; \ \forall \ k \in C
    \]
Contingency-constrained model

Issues

• Problem dimension depends on the number of contingencies

• Critical issue for tighter security criteria \((n - K)\) and/or large-scale systems \(\Rightarrow\) Computational intractability

• Need for new approaches
Contents

- Introduction
  - Adjustable robust optimization
- Solution methodology
- Numerical results
- Conclusions
Adjustable robust optimization (ARO)

- Optimization under a pre-specified uncertainty set
- Uncertain variables are allowed to vary within certain limits
- Uncertainty budget $\Rightarrow$ Limit on the conservativeness of the solution
- Worst-case optimization
ARO for contingency-constrained models

- $A^k_i$ and $A^k_l$ represent “uncertainty” and are respectively replaced by 0/1 decision variables $a^G_i$ and $a^L_l$ ⇒ Polyhedral uncertainty set

- Uncertainty budget ⇒ Security criterion:

$$f\left(\{a^G_i\}_{i \in I}, \{a^L_l\}_{l \in L}\right) \geq 0$$

- **Non-dependent on index $k$!!**
ARO for contingency-constrained models

- Contingency constraints are replaced by an optimization problem to characterize the worst case
- Worst case $\Rightarrow$ Maximum damage (system power imbalance) associated with \textbf{ALL} contingencies implicitly modeled by $a_i^G$ and $a_i^L$
- Robust counterpart $\Rightarrow$ Multilevel optimization (bilevel programming, trilevel programming)
Trilevel robust counterpart

- Upper-Level Problem
  - Least-cost pre-contingency state
  - Determine: Pre-contingency decisions

- Middle-Level Problem
  - Worst-case contingency
  - Determine: Unavailable components

- Lower-Level Problem
  - System damage minimization
  - Determine: Corrective actions
Trilevel robust counterpart

\[
\begin{align*}
\text{Minimize}_x & \quad c(x) + C^T \Delta^{wc}(x) \\
\text{subject to:} & \quad g(x) \leq 0 \\
\Delta^{wc}(x) &= \max_{a \in \{0,1\}} \delta(x, a) \\
\text{subject to:} & \quad f(a) \geq 0 \\
\delta(x, a) &= \min_{x^{wc}, \Delta^{wc}} (e^T \Delta^{wc}) \\
\text{subject to:} & \quad g^{wc}(x, a, x^{wc}, \Delta^{wc}) \leq 0
\end{align*}
\]
Contents

- Introduction
- Adjustable robust optimization
- Solution methodology
- Numerical results
- Conclusions
Solution methodology

- Two-step procedure:
  - Conversion of the trilevel program into an equivalent bilevel program
  - Application of Benders decomposition
- Guaranteed convergence to the optimal solution
Solution approach: Bilevel equivalent

- Two lowermost optimization levels (max-min problem) are transformed into an equivalent single-level mixed-integer linear program:
  - Dual of the lower-level problem renders the original max-min a max-max \( \Rightarrow \) Maximization problem (strong duality theorem)
  - Linearization of bilinear product terms
Bilevel equivalent w/o bilinear terms

\[
\begin{align*}
\text{Minimize}_x & \quad c(x) + C^l \Delta^{wc}(x) \\
\text{subject to: } & \quad g(x) \leq 0 \\
& \quad \Delta^{wc}(x) = \max_{a \in \{0,1\}, \pi, x^{aux}} h^{\text{dual-li}}(\pi, x, x^{aux}) \\
& \quad \text{subject to: } \\
& \quad f(a) \geq 0 \\
& \quad g^{\text{dual-li}}(\pi, a, x^{aux}) \leq 0
\end{align*}
\]
Nice features of the bilevel equivalent

- Lower-level problem $\Rightarrow$ Mixed-integer linear program parameterized in upper-level variables

- Upper-level decision variables $x$ only appear in linear terms in the lower-level objective function $\Rightarrow \Delta^{wc}(x)$ is a convex function of upper-level variables
Benders decomposition

- Worst-case power imbalance $\Delta^{wc}(x) = \text{Recourse function}$

- Partial subgradients of $\Delta^{wc}(x)$ straightforwardly available $\Rightarrow$ Approximation of $\Delta^{wc}(x)$ via cutting planes (Benders cuts)

- Accelerated convergence $\Rightarrow$ Additional constraints in the master problem (column-and-constraint generation algorithm)
Contents

- Introduction
- Adjustable robust optimization
- Solution methodology
- Numerical results
- Conclusions
Test systems

- IEEE 118-bus system, IEEE 300-bus system
- $K \leq 5$
- Increasing levels of system power imbalance are allowed for $K \geq 3$
- 0.001% optimality gap
- 2 Intel Xeon E5-2687W processors at 3.1 GHz, 128 GB RAM, Xpress-MP 7.5, MOSEL
IEEE 118-bus case

<table>
<thead>
<tr>
<th>K</th>
<th>ARO-based approach</th>
<th>Contingency-dependent model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>System cost ($)</td>
<td>Time (s)</td>
</tr>
<tr>
<td>0</td>
<td>1.08E+08</td>
<td>0.22</td>
</tr>
<tr>
<td>1</td>
<td>1.10E+08</td>
<td>8.56</td>
</tr>
<tr>
<td>2</td>
<td>4.18E+08</td>
<td>2261.99</td>
</tr>
<tr>
<td>3</td>
<td>1.60E+11</td>
<td>26.76</td>
</tr>
<tr>
<td>4</td>
<td>2.40E+11</td>
<td>46.00</td>
</tr>
<tr>
<td>5</td>
<td>2.40E+11</td>
<td>716.97</td>
</tr>
</tbody>
</table>

*Unfinished (optimality gap = 32.34%)
## IEEE 118-bus case

<table>
<thead>
<tr>
<th>$K$</th>
<th>$\Delta$ (%)</th>
<th>System power imbalance (%)</th>
<th>Number of circuits built</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.00</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0.00</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.00</td>
<td>33</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>3.10</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>4.65</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>4.65</td>
<td>8</td>
</tr>
</tbody>
</table>
## IEEE 300-bus case

<table>
<thead>
<tr>
<th>$K$</th>
<th>ARO-based approach</th>
<th>Contingency-dependent model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>System cost ($)</td>
<td>Time (s)</td>
</tr>
<tr>
<td>0</td>
<td>2.06E+09</td>
<td>0.78</td>
</tr>
<tr>
<td>1</td>
<td>2.08E+09</td>
<td>108.50</td>
</tr>
<tr>
<td>2</td>
<td>3.26E+09</td>
<td>7222.27</td>
</tr>
<tr>
<td>3</td>
<td>6.18E+11</td>
<td>1125.91</td>
</tr>
<tr>
<td>4</td>
<td>1.64E+12</td>
<td>188.49</td>
</tr>
<tr>
<td>5</td>
<td>2.28E+12</td>
<td>66.61</td>
</tr>
</tbody>
</table>
IEEE 300-bus case

<table>
<thead>
<tr>
<th>$K$</th>
<th>$\bar{\Delta}$ (%)</th>
<th>System power imbalance (%)</th>
<th>Number of circuits built</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.00</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0.00</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.00</td>
<td>46</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>1.26</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>3.35</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>4.65</td>
<td>4</td>
</tr>
</tbody>
</table>
Contents

• Introduction

• Adjustable robust optimization

• Solution methodology

• Numerical results

• Conclusions
Concluding remarks

- Robust optimization is suitable for deterministic contingency-constrained transmission expansion planning

- Implicit consideration of contingencies yields significant computational advantages over contingency-dependent models

- Adjustable robust optimization paves the way for the exploration of tighter security criteria
Further research

- Incorporation of uncertainty sources (demand, renewable power generation) ⇒ Impact of correlation

- Consideration of more sophisticated operational models (ac load flow, line switching)

- Analysis of alternative solution approaches to avoid the dual-based transformation
Reference

Thanks for your attention!!