Adaptive Robust Optimization with Dynamic Uncertainty Sets for Power System Operation Under Wind

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1. Power Systems Operation under Uncertainty
   1.1 Current Practice
   1.2 Existing Models

2. Our Proposal for Real-Time Operation under Significant Wind
   2.1 Dynamic Uncertainty Sets for Wind Power
   2.2 Two-Stage Robust ED and Rolling Horizon
   2.3 Computational Experiments
Electric power system is the backbone of modern society

Electric power networks are among the world’s most complex engineering systems
At a distance, it looks like this
Electric Power System

• Close up, it looks like this
• Graphically, it looks like this
Mathematically, it looks like this

\[
\begin{align*}
\min_{e, f, p_i^g, q_i^g} & \quad c(p_i^g) = \sum_{i \in \mathcal{G}} \left( c_{i0}(p_i^g)^2 + c_{i1}p_i^g + c_{i2} \right) \\
\text{s.t.} & \quad p_i^g - p_i^d = \sum_{j \in \mathcal{N}} \left( e_i(G_{ij}e_j - B_{ij}f_j) + f_i(G_{ij}f_j + B_{ij}e_j) \right), \quad \forall i \in \mathcal{N} \\
& \quad q_i^g - q_i^d = \sum_{j \in \mathcal{N}} \left( f_i(G_{ij}e_j - B_{ij}f_j) - e_i(G_{ij}f_j + B_{ij}e_j) \right), \quad \forall i \in \mathcal{N} \\
& \quad (e_i^2 + f_i^2)\tilde{G}_{ii} + (e_i e_j + f_i f_j)\tilde{G}_{ij} - (e_i f_j - e_j f_i)\tilde{B}_{ij} \leq p_{ij}^{\max}, \forall (i, j) \in \mathcal{L} \\
& \quad (v_i^{\min})^2 \leq e_i^2 + f_i^2 \leq (v_i^{\max})^2, \quad \forall i \in \mathcal{N} \\
& \quad p_i^{\min} \leq p_i^g \leq p_i^{\max}, \quad \forall i \in \mathcal{G} \\
& \quad q_i^{\min} \leq q_i^g \leq q_i^{\max}, \quad \forall i \in \mathcal{G},
\end{align*}
\]
Electric Power Systems Problems

• Key problems:

  – Different time scales from min to decades
  – Multiple agents (GenCo, TransCo, DistCo, ISO, Utility...)
  – Significant uncertainties (load, generation, outages, construction, ...)

- Generation Transmission Planning: 5-15 years
- Generation Transmission Maintenance: one year
- Day-Ahead Unit Commitment: 24 hours
- Real-Time Economic Dispatch: Every 5 min
• Day-ahead unit commitment & Real-time economic dispatch

Info: Supply costs, load forecast
Decision: which units to commit
Goal: meet demand w. min cost
Constraints: physical, security

Hour
-12
Day-ahead UC

Info: Unit commit, realized load
Decision: generation level
Goal: min costs meet demand
Constraints: physical, security

0
Real-time Dispatch
New Challenge: Growing Uncertainty

- New challenge

Supply Variation: Wind Power Penetration
40% annual growth

Net Load Uncertainty Can be Huge!

[ Ruiz, Philbrick 10 ]
Current Practice: Reserve Adjustment

• **UC**: Deterministic Model with Reserve
  Incorporating extra resources called reserve
  [Sen and Kothari 98] [Billinton and Fotuhi-Firuzabad 00]

• **ED**: Deterministic Single-Period Model

**Drawbacks:**
1. Uncertainty not explicitly modeled
2. Both system and locational requirement are preset, heuristic
3. Static model
Existing Proposals

• **UC: Stochastic Optimization Approach**
  Uncertainty modeled by distributions and scenarios
  [Takriti et. al. 96, 00] [Ozturk et. al. 05][Wong et. al. 07]

• **ED: Deterministic Multi-period (Look-Ahead)**

Weakness:
1. Hard to select “right” scenarios in large systems
2. Computational burden
3. Uncertainty still not considered in Look-Ahead ED
Existing Robust Models for UC and ED

- Robust Optimization for unit commitment
  - Adaptive two-stage robust SCUC models
    - [Jiang et. al. 2012], [Zhao, Zeng 2012],
    - [Bertsimas, Litvinov, Sun, Zhao, Zheng 2013] (joint w. ISO-NE)
  - RO for security optimization
    - [Street et. al. 2011], [Wang et. al. 2013]
  - Unifying RO with Stochastic UC
    - [Wang et. al. 2013]

- Robust Optimization for economic dispatch
  - AGC control (two-stage: dispatch + AGC)
    - [Zheng et. al. 2012]
  - Affine policy (dispatch as linear function of total load)
    - [Jabr 2013]
  - Robust multi-period ED for system flexibility
    - [Thattle, Sun, Xie 2014]
Common Features of Existing Robust Models

1. Similar Two-Stage Structures:
   • First-stage decision over multiple periods
   • Second-stage recourse over same periods

2. Static Uncertainty Sets
   • Do not capture correlations/dynamics of uncertainty processes
Uncertainty model of net load variation

$$\mathcal{D}^t(\tilde{d}^t, \hat{d}^t, \Delta^t) := \left\{ d^t \in \mathbb{R}^{N_d} : \sum_{i \in N_d} \frac{|d_i^t - \tilde{d}_i^t|}{\hat{d}_i^t} \leq \Delta^t, \right. $$

$$ \left. d_i^t \in [\tilde{d}_i^t - \hat{d}_i^t, \tilde{d}_i^t + \hat{d}_i^t], \forall i \in N_d \right\} $$

Budgeted
Two-Stage Fully-Adaptive Robust Optimization

• Adaptive Robust UC [Bertsimas et. al. 2013]
  – **Objective**: Fixed-Cost + Worst case Dispatch Cost

\[
\min_{x,u,v} \sum_{t} \sum_{i} F_i^t x_i^t + S_i^t u_i^t + G_i^t v_i^t + \max_{d \in \mathcal{D}} \min_{p \in \mathcal{W}(x,d)} \sum_{t} \sum_{i} C_i^t p_i^t
\]

\[s.t. \quad F(x, u, v) \leq 0\]
\[x, u, v \text{ binary.}\]

Constraints on commitment decision: Startup/shutdown, Min-up/down...

Find worst case \(d\) for dispatch

For a fixed \(x, d\) minimize dispatch cost

Second-Stage Problem
Our Proposal

- Dynamic Uncertainty Sets
- Two-Stage Robust ED and Rolling Horizon Policy
In a multi-period problem:

Let $\xi_t$ be the uncertainty vector at time $t$.

Uncertainty set of $\xi_t$ depends on the realization of uncertainties before $t$.

$$\Xi_t(\xi_{[1:t-1]}) = \left\{ \xi_t : \exists u_t \text{ s.t. } f(\xi_t, u_t) \leq 0 \right\}$$

For example: a dynamic interval uncertainty set:

$$\xi_t \in \left[ \underline{\xi}_t(\xi_{t-1}), \bar{\xi}_t(\xi_{t-1}) \right]$$

Polyhedral dynamic uncertainty sets:

$$\sum_{\tau=1}^{t} (A_{\tau} \xi_{\tau} + B_{\tau} u_{\tau}) \leq 0$$
A dynamic uncertainty set for wind speed:

\[ \mathcal{R}_t(r_{[t-L:t-1]}) = \left\{ r_t : \exists \tilde{r}_{[t-L:t]}, u_t \quad \text{s.t.} \quad r_t = g_t + \tilde{r}_t, \forall t = t - L, \ldots, t \right\}, \]

Seasonal pattern

Residual

Linear dynamics: Temporal & Spatial correlation

Uncertainty in Estimation with Budget Constraints
Wind Turbine Power Curve

- Power-curve model: GE 1.5MW wind turbine

### Technical data

<table>
<thead>
<tr>
<th>Operating Data</th>
<th>1.5sle</th>
<th>1.5xle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Capacity</td>
<td>1.500 kW</td>
<td>1.500 kW</td>
</tr>
<tr>
<td>Temperature Range</td>
<td>-30°C to +40°C</td>
<td>-30°C to +40°C</td>
</tr>
<tr>
<td>(with Grid-Weather Extreme Package)</td>
<td>-40°C to +50°C</td>
<td>-40°C to +50°C</td>
</tr>
<tr>
<td>Cut-in Wind Speed</td>
<td>3.5 m/s</td>
<td>3.5 m/s</td>
</tr>
<tr>
<td>Cut-out Wind Speed (10 min avg)</td>
<td>25 m/s</td>
<td>20 m/s</td>
</tr>
<tr>
<td>Rated Wind Speed</td>
<td>14 m/s</td>
<td>11.5 m/s</td>
</tr>
<tr>
<td>Wind Class — IEC</td>
<td>IIa $V_{e50} = 55$ m/s $V_{ave} = 8.5$ m/s</td>
<td>IIIb $V_{e50} = 52.5$ m/s $V_{ave} = 8.0$ m/s</td>
</tr>
</tbody>
</table>

### Electrical Interface

- Frequency: 50/60 Hz
- Voltage: 690V

### Rotor

- Diameter: 77 m
- Swept Area: 4657 m²

### Tower

- Hub Heights: 65/80 m

### Power Control

- Active Blade Pitch Control
• Use Power Curve to Model Available Wind Power:
  – Piecewise linear approx of turbine power curve:
\[
\overline{p}_{it}^{w} \geq h_{i0} + h_{ik} r_{it} \quad \forall k = 1, \ldots, K \tag{1}
\]
  Plateau part can be handled in optimization model
• Dynamic Uncertainty Set for Available Wind Power
\[
\mathcal{P}_{t}^{w}(\mathbf{r}_{[t-L:t-1]}) = \left\{ \overline{p}_{t}^{w} : \exists \mathbf{r}_{t} \in \mathcal{R}_{t}(\mathbf{r}_{[t-L:t-1]}) \text{s.t. (1) is satisfied} \right\}
\]
• Actual Power Dispatch from Wind Farms:
\[
0 \leq p_{it}^{w} \leq p_{i}^{w,\text{max}} \quad \forall i \in \mathcal{N}^{w}
\]
\[
p_{it}^{w} \leq \overline{p}_{it}^{w} \quad \forall i \in \mathcal{N}^{w}
\]
Two-Stage Robust ED and Rolling Horizon

• Adaptive robust ED:
  – **Time period 1** is decision to be implemented
  – **Future periods** with dynamic uncertainty sets

$$\min_{x \in \Omega_1^{det}} \left\{ c^\top x + \max_{d \in \mathcal{D}, \bar{p}^w \in \bar{P}^w} \min_{y \in \Omega(x, d, \bar{p}^w)} b^\top y \right\}$$

Stage-1

Stage-2

$t = 1:00$

$t = 1:05, 1:10, ..1:30$
• Rolling-horizon framework for real-time dispatch
• Dynamically update uncertainty model:
  – Wind uncertainty model updated every day and every 10 minutes
Experiment Setup

- IEEE Test Systems with 14-bus and 118-bus
- 14-bus system: 3 thermal gen, 4 wind farms, 11 loads, 20 transmission lines

Daily system demand:
132.6MW-319.1MW
Avg: 252.5MW
Wind farms:
- 4 wind farms, each of 75MW (50 GE 1.5MW)
- Real wind data: 5 min wind speed for a year
- Exhibit significant temporal/spatial correlations
- Avg wind speeds: 4.8m/s, 5.6m/s, 5.1m/s, 5.5m/s
- Avg total available wind power: 104.2MW
  - Equivalent to 34.7% capacity factor
  - Or 32.7% of peak load, 20% of thermal generation
  - Represent significant level of wind penetration
Objectives

• Adaptive robust ED v.s. Determ Look-Ahead ED:
  – Cost efficiency
  – Operational reliability
  – Insights on robust ED operation behavior

• Dynamic uncertainty sets v.s. Static uncertain. Sets:
  – Pareto frontier of cost-v.s.-reliability curve
• Adaptive robust ED v.s. Determ Look-Ahead ED:

<table>
<thead>
<tr>
<th>PERFORMANCE OF ROBUST AND DETERMINISTIC ED</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma$</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>0.0</td>
</tr>
<tr>
<td>0.1</td>
</tr>
<tr>
<td>0.3</td>
</tr>
<tr>
<td>0.5</td>
</tr>
<tr>
<td>0.7</td>
</tr>
<tr>
<td>1.0</td>
</tr>
</tbody>
</table>

- Cost Avg: Rob-ED 7.1% ($\Gamma = 0.5$) lower than LA-ED
- Cost StD: Rob-ED 41.2% lower than LA-ED; Rob-ED up to 82.0% lower than LA-ED
- Penalty freq: Rob-ED 52.4% lower than LA-ED; Rob-ED up to 80.1% lower than LA-ED
Robust ED Prepares System for Wind Drop

- Behavior of Rob-ED model:

<table>
<thead>
<tr>
<th>Operational Aspects of Robust and Deterministic ED</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>( \Gamma^{\psi} )</strong></td>
</tr>
<tr>
<td>-----------------------------</td>
</tr>
<tr>
<td>0.0</td>
</tr>
<tr>
<td>0.1</td>
</tr>
<tr>
<td>0.3</td>
</tr>
<tr>
<td>0.5</td>
</tr>
<tr>
<td>0.7</td>
</tr>
<tr>
<td>1.0</td>
</tr>
</tbody>
</table>

| Therm avg (MW) | 87.9 | 87.2 | 85.0 | 80.8 | 74.0 | 61.5 |
| Wind avg (MW)  | 87.9 | 87.2 | 85.0 | 80.8 | 74.0 | 61.5 |
Dynamic U Sets Pareto Dominate

- Dynamic uncertainty sets v.s. Static uncert sets

SUS1: No temp
SUS2: No temp/spatial
DUS: w. temp/spatial

Pareto Frontier
Wind Uncertainty is Most Important

- Load Uncertainty + Wind Uncertainty
### IEEE 118-Bus Test Results:

<table>
<thead>
<tr>
<th>Performance of LA-ED and Rob-ED for 118-bus system</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Gamma^w )</td>
</tr>
<tr>
<td>0.0</td>
</tr>
<tr>
<td>0.5</td>
</tr>
<tr>
<td>1.0</td>
</tr>
<tr>
<td>1.5</td>
</tr>
<tr>
<td>2.0</td>
</tr>
<tr>
<td>Cost Avg ($)</td>
</tr>
<tr>
<td>Cost Std ($)</td>
</tr>
<tr>
<td>Penalty Avg ($)</td>
</tr>
<tr>
<td>Penalty Freq (%)</td>
</tr>
<tr>
<td>Therm Avg (MW)</td>
</tr>
<tr>
<td>Wind Avg (MW)</td>
</tr>
</tbody>
</table>

- Cost Avg: Rob-ED reduces 43.4% (1.5), 39.7% (2.0)
- Cost StD: Rob-ED reduces 87.7% (1.5), 93.9% (2.0)
- Penalty freq: Rob-ED reduces 98.4% (1.5), 99.7% (2.0)

Summary

• New Uncertainty Models for Uncertain Sources with High Spatial/Temp Correlation
  – Dynamic Uncertainty Sets
  – Data Driven Approach

• Different Two-Stage Robust Models for Real-Time Operations
  – Rolling Horizon Market Operation

• Future direction:
  – Deeper understanding of dynamic uncertainty sets
  – Data-driven approach for uncertainty modeling
  – Market integration
  – Implementation in real systems
THANK YOU!

Questions?

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  ISyE, Georgia Tech
A Real-World Example: ISO-NE Power System

- 312 Generators
- 174 Loads
- 2816 Nodes
- 90 representative trans lines
- 24-hr data: gen/load/reserve
- Total gen cap: 31.4GW
- Total forecast load: 14.1GW
• Solve AdptRob and ResAdj UC solutions for \( \Delta^t = 0, 0.1Nd, \ldots, Nd \) for all \( t \).

• Fix UC solutions, simulate dispatch over load samples
  – 1000 load samples from \([\bar{d}_i^t - \hat{d}_i^t, \bar{d}_i^t + \hat{d}_i^t]\)

• Compute average dispatch cost and std.

• Avg dispatch cost: Economic efficiency
• Standard deviation: Price and Operation Stability
• Robustness to distributions
Computational Results (I-a): Average dispatch cost

Average Dispatch Cost under Normal Distribution

Average Dispatch Cost (M$)

Budget of Uncertainty (Δ/Nd)

- 2.7% relative saving or 472.9k$

Avg Dispatch Cost Relative Saving := (ResAdj – AdptRob)/ResAdj

0.65% - 2.7%
## Computational Results (II): Volatility of Costs

<table>
<thead>
<tr>
<th>Budget of Uncertainty</th>
<th>AdptRob Std disp cost ($k)</th>
<th>ResAdj Std disp cost ($k)</th>
<th>ResAdj/AdptRob</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>47.5</td>
<td>687.5</td>
<td>14.48</td>
</tr>
<tr>
<td>0.2</td>
<td>46.4</td>
<td>687.5</td>
<td>8.62</td>
</tr>
<tr>
<td>0.3</td>
<td>45.4</td>
<td>377.8</td>
<td>8.32</td>
</tr>
<tr>
<td>0.4</td>
<td>44.2</td>
<td>366.7</td>
<td>8.29</td>
</tr>
<tr>
<td>0.5</td>
<td>44.1</td>
<td>377.2</td>
<td>8.55</td>
</tr>
<tr>
<td>0.6</td>
<td>44.0</td>
<td>370.9</td>
<td>8.43</td>
</tr>
<tr>
<td>0.7</td>
<td>44.0</td>
<td>377.1</td>
<td>8.58</td>
</tr>
<tr>
<td>0.8</td>
<td>43.9</td>
<td>370.7</td>
<td>8.44</td>
</tr>
<tr>
<td>0.9</td>
<td>43.9</td>
<td>357.9</td>
<td>8.15</td>
</tr>
<tr>
<td>1.0</td>
<td>43.9</td>
<td>361.0</td>
<td>8.22</td>
</tr>
</tbody>
</table>

Coeff Var: $44k/17.2M=0.25\%$

370k/17.3M=2.1%

**Significant reduction in cost volatility!**
Computational Results (III): Robustness to Distribution

**Avg Dispatch Cost of AdptRob**

- **Uniform**
- **Normal**

Relative difference: 0.0368% - 0.0920%
Absolute difference: $6.3k – $15.8k
Computational Results: Robustness to Distribution

Avg Dispatch Cost of ResAdj

Relative difference: 1.00% - 2.19%
Absolute difference: $174.4k – $382.2k
Advantages of Adaptive Robust UC

- saves dispatch cost (6.92% $1.27M)
- significantly reduces cost volatility
- robust against load distributions

Economic Efficiency

Reduces Price & System Operation Volatility

Data Driven Approach Demand Modeling