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I. Introduction

Electricity Deregulation

Load and Distributed Generators

Stable supply of electricity

Blackouts
1987 August: TOKYO
2003 August: United States, Canada
September: Sweden, Denmark, Italy

Increase of reactive power

Load Control based on Load Analysis
One-machine one-load system

Load Composition and Voltage Stability Analysis

Generator

- System P-V Curve
- System Q-V Curve

Load

- Observed data V, P, Q, ...
- Load P-V Curve
- Load Q-V Curve

Considering Dynamic Load

Research Flow

Power Grid

Load Composition and Voltage Stability Analysis
Ⅱ. Power Flow in Power Grid

Looking over the upper system at a certain load-bus

The upper system

Fig. 1 Power grid
Power Flow

Fig. 2 Power flow at a load-bus

\[ \dot{V}_1 = V_1 e^{j\delta_1} \]

\[ \dot{V}_2 = V_2 e^{j\delta_2} \]

\[ \dot{V} = V e^{j\delta} \]

\[ \dot{I}_1 \]

\[ \dot{I}_2 \]

\[ \dot{I}_L \]

\[ \dot{I}_N \]

\[ \dot{P}_2, \dot{Q}_2 \]

\[ \dot{P}_N, \dot{Q}_N \]

\[ \dot{P}_L, \dot{Q}_L \]

The upper system

A certain load
Power Flow

\[ P_L + jQ_L = \dot{V} \dot{I}_L \]
\[ = \dot{V} \left( i_1 + i_2 + \ldots + i_N \right) \]
\[ = \dot{V} \left( \frac{\dot{V}_1 - \dot{V}}{\dot{Z}_1} + \frac{\dot{V}_2 - \dot{V}}{\dot{Z}_2} + \ldots + \frac{\dot{V}_N - \dot{V}}{\dot{Z}_N} \right) \]
\[ = \dot{V} \cdot \left( \frac{1}{\dot{Z}_1} \dot{V}_1 + \frac{1}{\dot{Z}_2} \dot{V}_2 + \ldots + \frac{1}{\dot{Z}_N} \dot{V}_N \right) - \left( \frac{1}{\dot{Z}_1} + \frac{1}{\dot{Z}_2} + \ldots + \frac{1}{\dot{Z}_N} \right) \dot{V} \]
\[ = \dot{V} \cdot \left( \frac{\dot{V}_1 + \dot{V}_2 + \ldots + \dot{V}_N}{\dot{Z}_1 + \dot{Z}_2 + \ldots + \dot{Z}_N} \right) - \dot{V} \]
\[ = \dot{V} \cdot \left( \frac{1}{\dot{Z}_1 + \dot{Z}_2 + \ldots + \dot{Z}_N} \right) \]
\[ \equiv \dot{V} \left( \frac{\dot{V}_0 - \dot{V}}{\dot{Z}_0} \right) \]
\[ \dot{V}_0 = \sum_{k=1}^{N} \frac{\dot{V}_k}{\dot{Z}_k} / \sum_{k=1}^{N} \frac{1}{\dot{Z}_k} \]

Fig. 2 Power flow at a load-bus
The upper system can be rewritten into a one-machine one-load system without approximation.

\[ P_L + jQ_L \equiv \dot{V} \left( \frac{\dot{V}_0 - \dot{V}}{\dot{Z}_0} \right) , \]

\[
\begin{align*}
\dot{V}_0 & \equiv \sum_{k=1}^{N} \left( \frac{\dot{V}_k}{\dot{Z}_k} \right) \left/ \sum_{k=1}^{N} \left( \frac{1}{\dot{Z}_k} \right) \right. \\
\dot{Z}_0 & \equiv \frac{1}{\sum_{k=1}^{N} \left( \frac{1}{\dot{Z}_k} \right)}
\end{align*}
\]

Fig. 3 Equivalent one-machine one-load system
### III. Load Modeling

**Static Load Model**

- **Exponential-type**
  \[ P = P_0 V^n \]  
- **Polynomial-type**
  \[ P = a_0 + a_1 V + a_2 V^2 \]

**Dynamic Load Model**

- **Constant current**
- **Constant power**
- **Constant impedance**

\[ P_L(t) = a_0 + a_1 V_2(t) + a_2 \left( G(t) V_2^2(t) - 1 \right) \]  
\[ \frac{dG(t)}{dt} = -\frac{1}{T} \left( G(t) V_2^2(t) - 1 \right) \quad , \quad G(0) = 1 \]

Ihara, Tomiyama et al. (IEEE 1999)
Load Modeling

Fig. 4 Modeling of $V_2(t)$, $P_L(t)$ and $Q_L(t)$
Load Modeling \(\textcircled{2}\)

**Dynamic Load Model**

\[ P_L(t) = a_0 + a_1 V_2(t) + a_2 \left( G(t) V_2^2(t) - 1 \right) \]  

(5)

\[ \frac{dG(t)}{dt} = -\frac{1}{T} \left( G(t) V_2^2(t) - 1 \right) , \quad G(0) = 1 \]  

(6)

**Fig. 5 Conductance G(t)**

\[ T = 0.0167 \]
Load Modeling ③

Quadratic Function Model

\[ G(t) \approx m + nV_2(t) + pV_2^2(t) \]  

(7)

Fig. 6 Relationship between \( V_2(t) \) & \( G(t) \)
Load Modeling \(④\)

\[
P_L(t) = a_0 + a_1 V_2(t) + a_2 \left( G(t) V_2^2(t) - 1 \right) \quad (5)
\]

\[
\frac{dG(t)}{dt} = -\frac{1}{T} \left( G(t) V_2^2(t) - 1 \right) \quad (6)
\]

\[
G(t) \approx m + n V_2(t) + p V_2^2(t) \quad (7)
\]

Load P-V Curve

\[
P_L = (a_0 - a_2) + a_1 V_2 + a_2 m V_2^2 + a_2 n V_2^3 + a_2 p V_2^4 \quad (8)
\]

Load Q-V Curve

\[
Q_L = (b_0 - b_2) + b_1 V_2 + b_2 r V_2^2 + b_2 s V_2^3 + b_2 u V_2^4 \quad (9)
\]

Load P-V and Q-V Curves considering Dynamic Load
Load Modeling ⑤

Fig. 7 Load composition
IV. Voltage Stability Analysis

Fig. 8 The standard one-machine system model with load.

\[ \dot{V}_1 = V_1 \angle \delta_1 \]

\[ V_1 = 1 \text{ p.u.} \]

\[ 1:1.05 \]

\[ 1:1 \]

\[ R + jX \]

\[ \dot{V}_2 = V_2 \angle \delta_2 \]

\[ \dot{V}_2 = \frac{V_1 - V_2}{R_1 + jX_1} \]

\[ P_L + jQ_L = V_2 \left( \frac{\dot{V}_1 - \dot{V}_2}{R_1 + jX_1} \right) \]

(10)

\[ Q_L = P_L \tan \phi \]

(11)

\[ \left( R^2 + X^2 \right) \left( 1 + \tan^2 \phi \right) P_L^2 + 2 \left( R + X \tan \phi \right) V_2^2 \cdot P_L + \left( V_2^4 - V_1^2 V_2^2 \right) = 0 \]

(12)

(for \( \forall t \geq 0 \))
Fig. 9 System and Load P-V curves
Q-V curves

Fig. 10 System and Load Q-V Curves
Fig. 11 Sensitivity analysis
V. Smart Grid

1. Load Analysis (Real-Time Load Analysis)
2. Load Control
VI . Conclusion

(1) A power grid can be rewritten without approximation into an equivalent system at each load-bus.
(2) The dynamic load model with a differential equation can be approximated in a higher order polynomial form of $V_2$.
(3) The voltage stability can be analyzed by the system and the load P-V and Q-V curves considering dynamic load.
(4) In the case of the data observed in Sweden, the reactive power is more sensitive to the voltage stability than the active one and the case of Japan.

Future Subject

Applying the proposed method to the data observed by changing LTC at sub-station, and to the load control for Smart Grid.