Optimal and Autonomous Incentive-based Energy Consumption Scheduling Algorithm for Smart Grid

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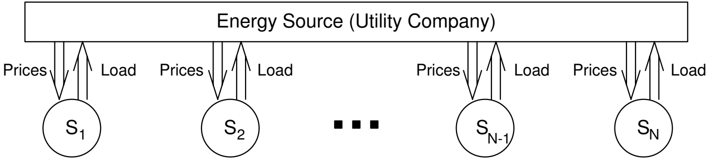
Outline

- Traditional Load Control vs. Load Control in Smart Grid
- System Model and Problem Formulation
- Energy Consumption Game
- Simulation Results
- Conclusions

Traditional Load Control



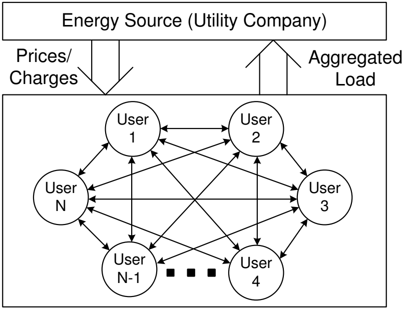
- Some variations:
  - Direct load control
  - Real-time Pricing
  - Conservation Rates
  - Etc.

- Key feature:
  - The focus of load control is on the interactions between the utility and each user.

What Does the Utility Care About?

- The utility cares about the aggregated behavior of all users:
  - The total load at each hour.
  - The peak-to-average ratio (PAR) in total load demand.
- Therefore, the current setting does not seem to be the best choice.

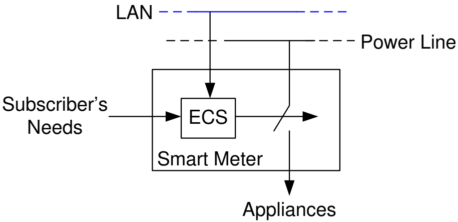
Load Control in Smart Grid



- In this setting, the users also interact with each other through message exchanges.
- Message exchanges can be:
  - Manual $\Rightarrow$ Phone, E-mail, etc.
  - Automatic over a two-way communication infrastructure $\Rightarrow$ "Smart Grid".

OUR FOCUS

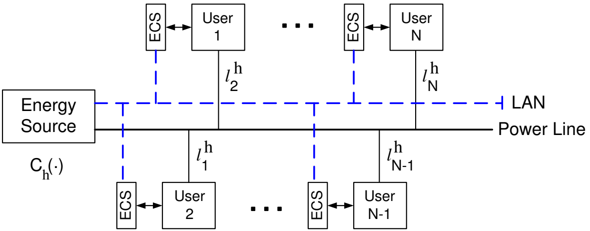
- To design an autonomous and incentive-based load control system:
  - We deploy energy consumption scheduling (ECS) units in "Smart Meters".
  - Users should have incentives to use ECS units ⇒ "Smart Pricing".



- ECS unit schedules energy consumption of all/most household appliances.

System Model: Single Shared Energy Source

- We consider a scenario where a source of energy is shared by several users, each one equipped with an ECS-enabled smart meter.



- Here, we have:
  - Energy source: generator, step-down substation transformer, etc.
  - l_n^h: Power load of user $n \in \mathcal{N} = \{1, \ldots, N\}$ at hour $h \in \mathcal{H} = \{1, \ldots, 24\}$.
  - $C_h(\sum_{n \in \mathcal{N}} l_n^h)$: Energy cost at hour $h \in \mathcal{H}$ as a function of total hourly load.

System Model: Household Appliances

- For each user $n \in \mathcal{N}$, the set of appliances is shown by $\mathcal{A}_n$.
- For each $a \in \mathcal{A}_n$, we define
  - $E_{n,a}$: pre-determined total daily energy consumption required.
- Example: $E_{n,a} = 9$ kWh for a PHEV for a 40-mile daily driving range.
- Clearly, we have

$$\sum_{h \in \mathcal{H}} l_n^h = \sum_{a \in \mathcal{A}_n} E_{n,a}, \qquad \forall\, n \in \mathcal{N}.$$

- We want to shift consumption to reduce PAR, not to reduce consumption!

System Model: Scheduling

- For each user $n \in \mathcal{N}$ and each appliance $a \in \mathcal{A}_n$, we define

$$\mathbf{x}_{n,a} \triangleq [x_{n,a}^1, \ldots, x_{n,a}^{24}],$$

  where $x_{n,a}^h$ denotes the scheduled one-hour energy consumption for appliance a.

- The ECS unit's job is to choose the right values for $\mathbf{x}_{n,a}$ for each $a \in \mathcal{A}_n$.
  - It should satisfy the users' needs.

System Model: User's Needs

- To address user n's needs, we further define
  - $\alpha_{n,a}$: Beginning of the time interval that consumption can be scheduled.
  - $\beta_{n,a}$: End of the time interval that consumption can be scheduled.
  - $\gamma_{n,a}^{\text{min}}, \gamma_{n,a}^{\text{max}}$: Minimum and maximum scheduled power levels.

- For each user $n \in \mathcal{N}$, we can define feasible scheduling set as follows:

$$\mathcal{X}_n = \Big\{ \mathbf{x}_n \mid \sum_{h=\alpha_{n,a}}^{\beta_{n,a}} x_{n,a}^h = E_{n,a}, \quad \forall\, a \in \mathcal{A}_n, \\ x_{n,a}^h = 0, \quad \forall\, h \in \{1, \ldots, \alpha_{n,a} - 1\} \cup \{\beta_{n,a} + 1, \ldots, 24\}. \\ \gamma_{n,a}^{\text{min}} \le x_{n,a}^h \le \gamma_{n,a}^{\text{max}}, \quad \forall\, h \in \{\alpha_{n,a}, \ldots, \beta_{n,a}\} \Big\}.$$

where $\mathbf{x}_n \triangleq [\mathbf{x}_{n,a}, \ \forall\, a \in \mathcal{A}_n]$. The ECS unit may only select $\mathbf{x}_n \in \mathcal{X}_n$.

System Model: Energy Cost

Question: Which is the best feasible choice of $\mathbf{x}_n$ for each user $n \in \mathcal{N}$?

- Consider the total load in the system at each hour of the day $h \in \mathcal{H}$:

$$L_h \triangleq \sum_{n \in \mathcal{N}} l_n^h = \sum_{n \in \mathcal{N}} \sum_{a \in \mathcal{A}_n} x_{n,a}^h,$$

- We define an energy cost function $C_h(L_h)$ indicating the cost of generating or providing energy by the energy source at each hour $h \in \mathcal{H}$.
- In general, the energy cost function can be different at different hours of the day.
- For example, the cost can be less at night compared to the day time.
- The cost can be actual or artificial, i.e., for load shaping.

System Model: Energy Cost

- We assume that the energy cost functions are increasing and convex

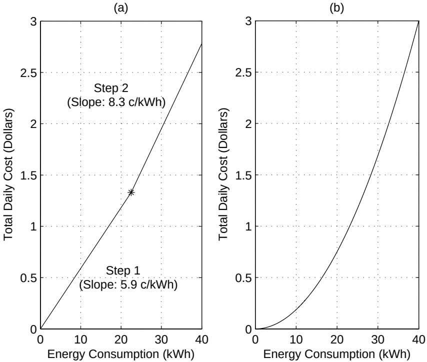


Figure: Two sample convex and increasing cost functions: (a) Two-step conservation rate model used by BC Hydro; (b) A quadratic cost function forming a similar shape.

Optimization Problem

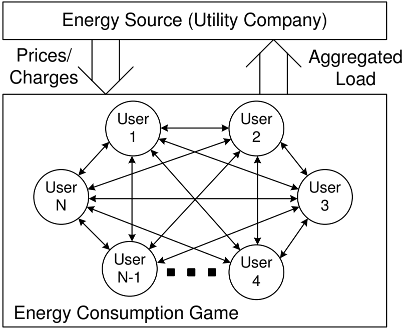
- Given complete knowledge about the users' needs and a centralized control of the system, an efficient energy consumption scheduling to be implemented by the ECS units can be characterized as the solution of the following problem:

$$\begin{aligned} \underset{\mathbf{x}_1,\ldots,\mathbf{x}_N}{\textbf{minimize}} \quad & \sum_{h=1}^{24} C_h\left(\sum_{n\in\mathcal{N}}\sum_{a\in\mathcal{A}_n} x_{n,a}^h\right) \\ \textbf{subject to} \quad & \mathbf{x}_n \in \mathcal{X}_n, \quad \forall\, n \in \mathcal{N}. \end{aligned} \tag{1}$$

- Problem (1) has several constraints which are encapsulated in sets $\mathcal{X}_n$.
- Problem (1) is a convex optimization problem.
- Problem (1) has a unique optimal solution if the cost function is strictly convex.

Design Objective

- Solve Problem (1) in a distributed fashion with cooperation from the users.
  - A good pricing model can encourage users to cooperate for their own benefits.



- The system can be modeled as an energy consumption game.

Pricing and Billing Model

- Let b_n denote what appears on the bill for user $n \in \mathcal{N}$ at the end of the day.
- In general, we would expect two properties to hold in the pricing model.
- **Property I** - The total daily charge should be as high as the total daily cost:

$$\sum_{n \in \mathcal{N}} b_n \geq \sum_{h=1}^{24} C_h \left(\sum_{n \in \mathcal{N}} l_n^h \right),$$

That is

$$\kappa \triangleq \frac{\sum_{n \in \mathcal{N}} b_n}{\sum_{h=1}^{24} C_h \left(\sum_{n \in \mathcal{N}} l_n^h \right)} \geq 1.$$

- If $\kappa = 1$, then the system is budget balanced. But usually, $\kappa > 1$.

- **Property II** - We have:

$$\frac{b_n}{b_m} = \frac{\sum_{h=1}^{24} l_n^h}{\sum_{h=1}^{24} l_m^h} = \frac{\sum_{a \in \mathcal{A}_n} E_{n,a}}{\sum_{a \in \mathcal{A}_m} E_{m,a}}, \quad \forall n, m \in \mathcal{N}.$$

- After reordering the terms, we have

$$\sum_{m \in \mathcal{N}} b_m = \sum_{m \in \mathcal{N}} \left(b_n \frac{\sum_{h=1}^{24} l_m^h}{\sum_{h=1}^{24} l_n^h} \right) = b_n \frac{\sum_{m \in \mathcal{N}} \sum_{h=1}^{24} l_m^h}{\sum_{h=1}^{24} l_n^h}.$$

Pricing and Billing Model

- There is only a single pricing model which satisfies Properties I and II:

$$
\begin{aligned}
b_n &= \frac{\sum_{h=1}^{24} l_n^h}{\sum_{m\in\mathcal{N}} \sum_{h=1}^{24} l_m^h} \left(\sum_{m\in\mathcal{N}} b_m \right) \\
&= \frac{\kappa \sum_{h=1}^{24} l_n^h}{\sum_{m\in\mathcal{N}} \sum_{h=1}^{24} l_m^h} \left(\sum_{h=1}^{24} C_h \left(\sum_{m\in\mathcal{N}} l_m^h \right) \right) \\
&= \frac{\kappa \sum_{a\in\mathcal{A}_n} E_{n,a}}{\sum_{m\in\mathcal{N}} \sum_{a\in\mathcal{A}_m} E_{m,a}} \left(\sum_{h=1}^{24} C_h \left(\sum_{m\in\mathcal{N}} \sum_{a\in\mathcal{A}_m} x_{m,a}^h \right) \right).
\end{aligned}
$$

- The charge to one user also depends on other users schedules!
- That explains why it is a game among users.

ENERGY CONSUMPTION GAME

We can formally define the energy consumption game among the users as:

- ***Players***: Registered users in set $\mathcal{N}$.
- ***Strategies***: Energy consumption scheduling vectors $\mathbf{x}_n$ for all users.
- ***Payoffs***: $P_n(\mathbf{x}_n; \mathbf{x}_{-n})$ for each user $n \in \mathcal{N}$, where

$$P_n(\mathbf{x}_n; \mathbf{x}_{-n}) = -b_n$$
$$= -\frac{\kappa \sum_{a \in \mathcal{A}_n} E_{n,a}}{\sum_{m \in \mathcal{N}} \sum_{a \in \mathcal{A}_m} E_{m,a}} \left(\sum_{h=1}^{24} C_h \left(\sum_{m \in \mathcal{N}} \sum_{a \in \mathcal{A}_m} x_{m,a}^h \right) \right).$$

Here, $\mathbf{x}_{-n}$ denotes the vector of scheduling variables for all users other than user n.

Best Response

- Given $\mathbf{x}_{-n}$, each user $n \in \mathcal{N}$ tries to maximize its payoff:

$$\underset{\mathbf{x}_n \in \mathcal{X}_n}{\textbf{maximize}} \quad P_n(\mathbf{x}_n; \mathbf{x}_{-n}). \tag{2}$$

- This is equivalent to the following:

$$\underset{\mathbf{x}_n \in \mathcal{X}_n}{\textbf{minimize}} \quad \sum_{h=1}^{24} C_h \left(\sum_{a \in \mathcal{A}_n} x_{n,a}^h + \sum_{m \in \mathcal{N} \setminus \{n\}} \sum_{a \in \mathcal{A}_m} x_{m,a}^h \right).$$

- User/player n can only change the blue part.
- Playing according to (2) is called best response play.

Nash Equilibrium

- Nash equilibrium is a solution concept in game theory.
- The energy consumption scheduling vectors ($\mathbf{x}_n^*, \ \forall n \in \mathcal{N}$) form a Nash equilibrium if and only if for each user/player $n \in \mathcal{N}$,

$$P_n(\mathbf{x}_n^*; \mathbf{x}_{-n}^*) \geq P_n(\mathbf{x}_n; \mathbf{x}_{-n}^*), \quad \forall \ \mathbf{x}_n \geq 0.$$

- No user/player would try to deviate from a Nash equilibrium.
- Questions to be answered:
  - Does Nash equilibrium always exist? $\Rightarrow$ Yes!
  - Is it unique? $\Rightarrow$ Yes!
  - Do we converge from any initial point to Nash equilibrium? $\Rightarrow$ Yes!
  - What is the performance at Nash equilibrium? $\Rightarrow$ Optimal!

Nash Equilibrium

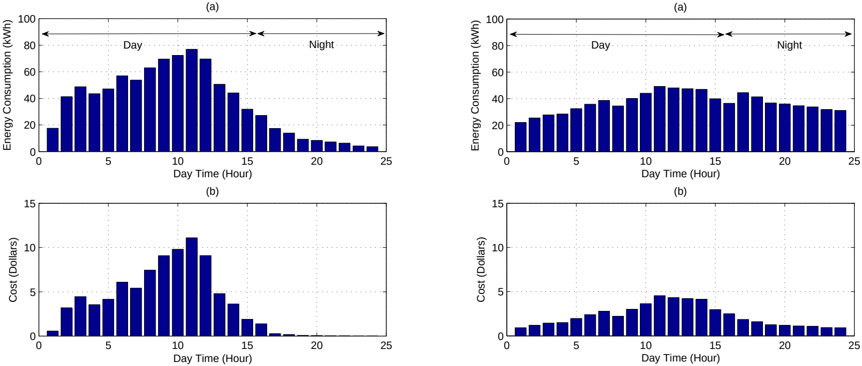
- Therefore, to solve problem (1) in a distributed fashion:
  - It is enough to use the pricing model explained earlier.
  - The users will automatically find the optimal solution in a game setting.
- Another key difference with traditional load control:
  - Here, the utility does not announce the exact price.
  - Instead, it announces the function that determines the price.

SIMULATION RESULTS: SETTING

Consider the following simulation setting:

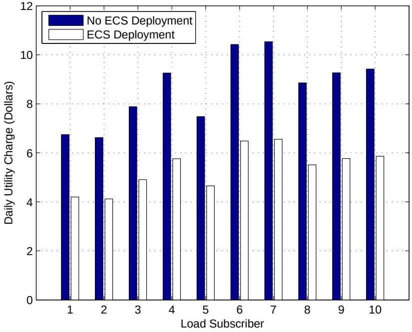
- The number of users is 10.
- Appliances with **hard** energy consumption scheduling constraints:
  - Refrigerator-freezer (daily usage: 1.32 kWh), electric stove (daily usage: 1.89 kWh for self-cleaning and 2.01 kWh for regular), lighting (daily usage for 10 standard bulbs: 1.00 kWh), heating (daily usage: 7.1 kWh), etc.
- Appliances with **soft** energy consumption scheduling constraints:
  - Dishwasher (daily usage: 1.44 kWh), clothes washer (daily usage: 1.49 kWh for energy-star 1.94 kWh for regular), clothes dryer (daily usage: 2.50 kWh), and PHEV (daily usage: 9.9 kWh), etc.
- The ECS units schedule consumption only for appliances with soft constraints.
- Cost function is quadratic with half price at night.

SIMULATION RESULTS: COST AND PAR



- **Left**: Results without using ECS units, PAR = 2.1, Cost = $86.47.
- **Right**: Results when we use ECS units, PAR = 1.3, Cost = $53.81.

SIMULATION RESULTS: INDIVIDUAL CHARGES



- Every user will benefit from using ECS units.
- Users will have the incentives to participate in ECS program.

CONCLUSIONS

- In Smart Grid, load control is an energy consumption game among users.
- The utility may only announce price functions not exact price values.
- Distributed best response plays will automatically lead to optimal performance.
- The right choice of the pricing model/function is the key.
- Simulation results show reductions in energy cost and peak-to-average-ratio.