Reliability-based Sizing of Backup Storage

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***Abstract*—This letter describes an analytical approach to determining the size, in terms of both power and energy capacity, of a backup storage unit in such a way as to meet a specified reliability target. The backup could be in the form of electrical energy storage or fuel storage. The proposed approach might benefit facilities that require high levels of reliability in their electric supply.**

***Index Terms*—storage, storage sizing, backup storage, reliability, availability, reliability target.**

I. Introduction

IT is common practice to use storage devices to provide service of higher reliability to loads that are considered critical. In this letter the term 'storage' is used in a general sense, to include both electrical storage (such as a battery, possibly as part of an uninterruptible power supply or UPS), and fuel (such as diesel, such as that stored in the tank of a diesel generator). In determining the appropriate size of backup storage for a particular application, one generally selects the power rating on the basis of the load to be supported, and the energy rating on the basis of the period of time one expects to have to support the load in the event of a failure of the primary supply. The energy rating on UPS units is often provided in Ampere-hours (Ah); in case of fuel storage, the size of the tank often determines the energy capacity, but this can often be supplemented with additional storage on site.

This letter presents a method of selecting the size of a storage device in such a manner as to meet a reliability target. This method of sizing backup storage equipment might be considered appropriate for facilities that have loads requiring very high levels of supply reliability.

II. Approach

Consider a system that is supplied by a primary source (such as a utility) of availability A_0. Here, *availability* implies the steady state probability that power will be served to the load [1]. Assume that of the system load there is a part P_C (where P_C is the power requirement) that is considered critical. It is intended to add sufficient storage to increase the availability of power to this critical load to A_1. It is clear that the power capacity of the required storage unit should be at least P_C. What remains to be determined is the energy capacity.

Define the fraction α such that

$$\alpha = \frac{1 - A_1}{1 - A_0}. \tag{1}$$

This fraction α, also known as the *unavailability reduction ratio*, can be understood as follows. Suppose $A_0 = 0.9999$ and the intention is to increase the availability of power to the critical load by 'an additional 9', i.e., to $A_1 = 0.99999$; then

$$\alpha = \frac{0.00001}{0.0001} = 0.1.$$

In developing the method for determining the required energy capacity, consider the following analysis. Denote

- S_F event that the primary supply has failed;
- L event that the critical load experiences failure of power supply;
- t_A length of time for which the storage unit can support the critical load in the event of failure of the primary supply;
- R random variable representing the down time (outage duration) of the primary supply;
- $f_R(r)$ probability density function of R.

Then the event L occurs when the primary supply is down for a period longer than t_A, and its probability is given by

$$\begin{aligned} P\{L\} &= P\{\{R > t_A\} \cap S_F\} \\ &= P\{R > t_A | S_F\} P\{S_F\} \\ &= \left(\int_{t_A}^{\infty} f_R(r) dr \right) P\{S_F\}. \end{aligned} \tag{2}$$

In (2), $P\{L\}$ is clearly $1 - A_1$ and $P\{S_F\} = 1 - A_0$. It is also clear from (1) and (2) that

$$\int_{t_A}^{\infty} f_R(r) dr = \alpha. \tag{3}$$

Equation (3) forms the basic relationship from which the required storage capacity can be determined. However, the storage device itself may have a certain probability of failure. In order to compensate for this, the storage device should have an energy capacity that enables it to provide the required power (P_C) for a period of time t_S that is given by

$$t_S = \frac{t_A}{A_S}, \tag{4}$$

where t_A is given by (3) and A_S is the availability of the storage device. So the power capacity of the selected storage unit should be at least P_C, and the energy capacity should be at least $P_C t_S$.

III. Solution of the Integral Equation

In many instances, the failure rate of the primary supply is constant, and the down time R is exponentially distributed [1]. For exponentially distributed R, the probability density function can be expressed in the form

$$f_R(r) = \frac{1}{\bar{r}} \exp\left(-\frac{r}{\bar{r}}\right), \quad r \geq 0, \tag{5}$$

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where $\bar{r}$ is the expectation (mean) of R. Then the solution of the integral equation (3) provides

$$t_A = -\bar{r} \ln \alpha. \quad (6)$$

So, for instance, if one were to use $\alpha = 0.1$ as in the example above, then a reasonably reliable storage unit ($A_S = 0.9$) would have to be capable of supporting the critical load for about 2.5 times the mean down time of the primary supply.

In instances where the failure rate is not constant, R follows other distributions, mostly Weibull or lognormal [1].

For a Weibull distribution with shape parameter β, the probability density function can be expressed in the form [1]

$$f_R(r) = \frac{\beta r^{\beta-1}}{r'^{\beta}} \exp\left[-\left(\frac{r}{r'}\right)^{\beta}\right], \quad r \geq 0, \quad (7)$$

where $r' = \bar{r}/\Gamma\left(1 + 1/\beta\right)$, and t_A can be obtained from

$$t_A = r' \left(-\ln \alpha\right)^{1/\beta}. \quad (8)$$

Note that for $\beta = 1$ the Weibull distribution degenerates to the exponential, and this is borne out by (7) and (8).

When R follows a lognormal distribution, there is no closed form solution. In case of R following a lognormal distribution with shape parameter β, the probability density function of R may be expressed in the form [1]

$$f_R(r) = \frac{1}{\sqrt{2\pi}\beta r} \exp\left[-\frac{1}{2\beta^2}\left(\frac{\beta^2}{2} + \ln\frac{r}{\bar{r}}\right)^2\right], \quad r \geq 0. \quad (9)$$

Then the solution of the integral equation given by (3) is obtained from

$$\Phi\left(\frac{1}{\beta}\left[\frac{\beta^2}{2} + \ln\frac{t_A}{\bar{r}}\right]\right) = 1 - \alpha, \quad (10)$$

where the function Φ is the cumulative distribution function of the standard normal variate, i.e.,

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-t^2/2)dt.$$

In order to solve (10), one would refer to a table of the standard normal distribution to look up the value of z corresponding to $\Phi(z) = 1 - \alpha$, and then determine t_A from

$$\frac{1}{\beta}\left(\frac{\beta^2}{2} + \ln\frac{t_A}{\bar{r}}\right) = z \Rightarrow t_A = \bar{r} \exp\left(\beta z - \frac{\beta^2}{2}\right). \quad (11)$$

IV. An Alternative Reliability Target

Instead of defining a reliability target in terms of availability, one may prefer to use a target in terms of mean down time. For instance, one may desire to reduce the mean down time of supply to the critical load to $\alpha\bar{r}$, where $\bar{r}$ is the mean down time of the primary supply. Then t_A, the length of time for which the storage unit should be able to support the critical load in the event of primary supply failure, is given by

$$\int_{t_A}^{\infty} r f_R(r)dr = \alpha\bar{r}. \quad (12)$$

It should be intuitively apparent that (3) and (12) would produce somewhat similar results; the proximity of these results will be clear from the following analysis. We know that

$$\bar{r} = \int_{0}^{\infty} r f_R(r)dr \Rightarrow \int_{0}^{\infty} (r - \bar{r}) f_R(r)dr = 0. \quad (13)$$

If one chose a value of t_A that satisfies (12), and this value were small, then from (13) it can be stated that

$$\int_{t_A}^{\infty} (r - \bar{r}) f_R(r)dr \approx 0 \quad \Rightarrow \quad \int_{t_A}^{\infty} r f_R(r)dr \approx \bar{r} \int_{t_A}^{\infty} f_R(r)dr$$
$$\Rightarrow \int_{t_A}^{\infty} f_R(r)dr \approx \alpha, \quad (14)$$

i.e., the t_A that satisfies (12) exactly also satisfies (3), approximately. In other words, if the primary supply were highly reliable, the two reliability targets would be approximately equivalent.

V. Concluding Remarks

This letter presented an analytical approach to determine the size of a backup storage unit so as to meet a specified reliability target. The size of a storage unit is defined by its power capacity and its energy capacity. The analysis presented applies to electrical storage as well as fuel storage, and can be used by any entity, such as a hospital, a process plant, or a military base, that is considering acquisition of electrical storage to meet an increased reliability target. The method can also be used by a facility that already has some standby generation or storage, to further increase the reliability; in such a case, the combination of utility and existing backup is treated as the primary supply, and the proposed method is used to determine the required capacity of additional storage.

The proposed analysis is applicable directly to determine capacities of new backup schemes that comprise single storage units. Where a backup scheme consists of multiple storage components, such as several UPS units, or standby generators, or a combination thereof, and where other factors are considered, such as maintenance of components, dependent or common mode failures, failures of generators to start, or time-dependent factors like state of charge, more involved methods [2]–[5] are necessary to determine energy capacities of the storage components.

References

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