A Flux-based Expression of Induction Machine Magnetizing Inductance

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***Abstract*—Tending towards the adaptive modeling of main flux saturation in induction machine, this brief is directed at disclosing the induction machine magnetizing inductance as an expression in terms of only *d-q* axis winding flux linkages. It will be shown that such an expression considerably facilitates the inclusion of main flux saturation effects in internal structure of the well-established winding flux linkage state-space model. The derivation procedure will be carried out without altering the structural equations of the generalized *d-q* axis mathematical model of induction machine.**

***Index Terms*—Induction machines, inductance, modeling.**

I. Introduction

WITH a view to dynamic security analysis [1], similar to most of electric power components, induction machine is usually described by a linear, reduced-order dynamic model. This is because the implementation of linear models of power system dynamics on the parallel or array processors makes the execution of a contingency analysis fast enough. However, with the increasing emphasis placed on economy, the induction machine is now operated in the immediate nearness of security limits and, thus, is exposed to large variations of magnetic stress. To formulate highly accurate dynamic *d-q* axis models, these variations have to be taken into account by means of the anhysteretic magnetizing curve of the machine [2]. Having this in view, this contribution is aimed at disclosing the induction machine magnetizing inductance as an expression in terms of only *d-q* axis winding flux linkages. Such an expression comes to be needful when the induction machine winding flux linkage state-space model is adopted that is when just all the *d-q* axis winding flux linkages are state variables. The adoption of the winding flux linkage state-space model is justified by the fact that the structural simplicity of this model is well-known [3].

II. The Induction Machine Generalized *D-Q* Axis Model

In a reference frame of velocity ω_{ref} with respect to the stator, the standard *d-q* axis mathematical model of induction machine is given by the following structural equations [3]:

(a) the voltage equations as ordinary differential equations:

$$(d\psi_{sd})/(dt) = \omega_{\text{ref}}\,\psi_{sq} - R_s\,i_{sd} + u_{sd}\,, \tag{1}$$

$$(d\psi_{sq})/(dt) = -\omega_{\text{ref}}\,\psi_{sd} - R_s\,i_{sq} + u_{sq}\,, \tag{2}$$

$$(d\psi_{rd})/(dt) = (\omega_{\text{ref}} - \omega_r)\cdot\psi_{rq} - R_r\,i_{rd}\,, \tag{3}$$

$$(d\psi_{rq})/(dt) = -(\omega_{\text{ref}} - \omega_r)\cdot\psi_{rd} - R_r\,i_{rq}\,, \tag{4}$$

wherein the symbols u, i, ψ denote voltages, currents and flux linkages, respectively, while subscripts s and r are associated with stator and rotor, respectively;

(b) the flux equations as algebraic correlations between *d-q* axis winding flux linkages and *d-q* axis winding currents:

$$\psi_{sd} = L_s\,i_{sd} + L_m\,i_{rd} = L_{s\sigma}\,i_{sd} + L_m\cdot(i_{sd} + i_{rd})\,, \tag{5}$$

$$\psi_{sq} = L_s\,i_{sq} + L_m\,i_{rq} = L_{s\sigma}\,i_{sq} + L_m\cdot(i_{sq} + i_{rq})\,, \tag{6}$$

$$\psi_{rd} = L_r\,i_{rd} + L_m\,i_{sd} = L_{r\sigma}\,i_{rd} + L_m\cdot(i_{sd} + i_{rd})\,, \tag{7}$$

$$\psi_{rq} = L_r\,i_{rq} + L_m\,i_{sq} = L_{r\sigma}\,i_{rq} + L_m\cdot(i_{sq} + i_{rq})\,, \tag{8}$$

where L_m stands for the magnetizing inductance variable, while index σ denotes the stator and rotor leakage inductances.

Notice that rotor quantities are referred to stator. The torque (motion) equation [3] is irrelevant here and is hence omitted.

III. Magnetizing Curve Representation

The effects of the main flux saturation in induction machine are accurately accounted for by means of machine anhysteretic magnetizing curve, described by the non-linearity [2], [3]:

$$\psi_m(i_m) = L_m(i_m)\cdot i_m$$

where the magnetizing flux ψ_m and magnetizing inductance L_m change to functions of magnetizing current i_m that is:

$$i_m = \sqrt{i_{md}^2 + i_{mq}^2} = \sqrt{(i_{sd} + i_{rd})^2 + (i_{sq} + i_{rq})^2}\,; \tag{9}$$

$$i_{md} = i_{sd} + i_{rd}\,, \quad i_{mq} = i_{sq} + i_{rq}\,.$$

With the purpose of magnetizing curve representation, a good compromise between accuracy and the simplicity of expressing is attained by employing a rational fraction approximation, e.g.

$$\psi_m(i_m) = L_m(i_m)\cdot i_m = \frac{\alpha - L_p\,i_m}{\beta + i_m}\cdot i_m \tag{10}$$

with parameter

$$L_p = L_{s\sigma}L_{r\sigma}/(L_{s\sigma} + L_{r\sigma}) = \text{const.} \tag{11}$$

Eq. (10) yields the magnetizing inductance as function of the magnetizing current (9), i.e. as function of *d-q* axis currents:

$$L_m = L_m(i_m) = \psi_m(i_m)/i_m = (\alpha - L_p\,i_m)/(\beta + i_m)\,. \tag{12}$$

However, as already pointed, the derivation is redirected here at disclosing the saturation-dependent magnetizing inductance as expression in terms of only *d-q* axis winding flux linkages.

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IV. Magnetizing Inductance Expressing in Terms of Winding Flux Linkages

Solving the system (5), (7) in relation to *d*-axis currents and the system (6), (8) in relation to *q*-axis currents, respectively, the following currents-flux linkages correlations result:

$$i_{sd} = [L_m \cdot (\psi_{sd} - \psi_{rd}) + L_{r\sigma} \psi_{sd}] / \Delta_m \,, \tag{13}$$

$$i_{sq} = [L_m \cdot (\psi_{sq} - \psi_{rq}) + L_{r\sigma} \psi_{sq}] / \Delta_m \,, \tag{14}$$

$$i_{rd} = [L_m \cdot (\psi_{rd} - \psi_{sd}) + L_{s\sigma} \psi_{rd}] / \Delta_m \,, \tag{15}$$

$$i_{rq} = [L_m \cdot (\psi_{rq} - \psi_{sq}) + L_{s\sigma} \psi_{rq}] / \Delta_m \tag{16}$$

with system determinant varying with magnetizing inductance: $\Delta_m = \Delta_m(L_m) = (L_{s\sigma} + L_{r\sigma}) \cdot L_m + L_{s\sigma} L_{r\sigma}$.

Taking advantage of (13)-(16), we proceed now to replace the *d-q* axis currents in expression (9) of magnetizing current. The following relationships successively come forth:

$$i_{md} = i_{sd} + i_{rd} = (L_{r\sigma} \psi_{sd} + L_{s\sigma} \psi_{rd}) / \Delta_m(L_m),$$

$$i_{mq} = i_{sq} + i_{rq} = (L_{r\sigma} \psi_{sq} + L_{s\sigma} \psi_{rq}) / \Delta_m(L_m),$$

$$i_m = \frac{\lambda_{dq}}{L_m + L_{s\sigma} L_{r\sigma} / (L_{s\sigma} + L_{r\sigma})} = \frac{\lambda_{dq}}{L_m + L_p} \tag{17}$$

with λ_{dq} (in Weber) as flux-dependent quantity:

$$\lambda_{dq} = \frac{\sqrt{(L_{r\sigma} \psi_{sd} + L_{s\sigma} \psi_{rd})^2 + (L_{r\sigma} \psi_{sq} + L_{s\sigma} \psi_{rq})^2}}{L_{s\sigma} + L_{r\sigma}} \,. \tag{18}$$

On the other hand, from (12) we straightforwardly draw out the magnetizing current as function of magnetizing inductance:

$$i_m = i_m(L_m) = (\alpha - \beta L_m) / (L_m + L_p) \,. \tag{19}$$

Eq. (17) coupled with specific dependency (19) bring forth an auxiliary equation to be solved in relation to magnetizing inductance, being here the saturation-dependent parameter:

$$\lambda_{dq} / (L_m + L_p) = (\alpha - \beta L_m) / (L_m + L_p) \,. \tag{20}$$

The symbolical solving of (20) yields:

$$L_m = L_m(\lambda_{dq}) = (\alpha - \lambda_{dq}) / \beta \,. \tag{21}$$

Eq. (21) provides the magnetizing inductance as function of flux-dependent quantity (18), i.e. in terms of *d-q* axis winding flux linkages. Thus, the flux-based expression (21) acts similar to expression (12) that provides the magnetizing inductance as function of magnetizing current (9), i.e. in terms of the *d-q* axis currents. Nevertheless, with (21), the *d-q* axis winding currents (13)-(16) are preserved as functions of *d-q* axis winding flux linkages. Besides, since its evaluation involves but elementary algebraic operations, the flux-based expression (21) comes to be suitable for the adaptive modeling of main flux saturation when the *d-q* axis winding flux linkages are state variables.

V. Conjunction with Induction Machine Winding Flux Linkage State-space Model

The winding flux linkage state-space model of induction machine is derived just by selecting all *d-q* axis winding flux linkages as state variables. To formulate this model, it comes to be necessary to remove the *d-q* axis currents from voltage equations (1)-(4) by using currents-flux linkages correlations. Thus, we will replace the *d-q* axis winding currents in (1)-(4) by expressions (13)-(16) in which the magnetizing inductance is now provided by flux-based expression (21). The processed voltage equations provide the flux linkages derivatives:

$$\frac{d}{dt}\begin{bmatrix} \psi_{sd} \\ \psi_{sq} \\ \psi_{rd} \\ \psi_{rq} \end{bmatrix} = \begin{bmatrix} -\omega_1 & \omega_2 & \omega_3 & 0 \\ -\omega_2 & -\omega_1 & 0 & \omega_3 \\ \omega_4 & 0 & -\omega_5 & \omega_6 \\ 0 & \omega_4 & -\omega_6 & -\omega_5 \end{bmatrix} \begin{bmatrix} \psi_{sd} \\ \psi_{sq} \\ \psi_{rd} \\ \psi_{rq} \end{bmatrix} + \begin{bmatrix} u_{sd} \\ u_{sq} \\ 0 \\ 0 \end{bmatrix} \tag{22}$$

wherein the present values of the flux linkages coefficients are here decided by flux-dependent quantity (18), in accordance with Eq. (21), and rotor angular velocity ω_r, respectively:

$$\omega_1 = R_s \cdot [L_m(\lambda_{dq}) + L_{r\sigma}] / [(L_{s\sigma} + L_{r\sigma}) \cdot L_m(\lambda_{dq}) + L_{s\sigma} L_{r\sigma}],$$

$$\omega_3 = R_s \cdot L_m(\lambda_{dq}) / [(L_{s\sigma} + L_{r\sigma}) \cdot L_m(\lambda_{dq}) + L_{s\sigma} L_{r\sigma}],$$

$$\omega_4 = R_r \cdot L_m(\lambda_{dq}) / [(L_{s\sigma} + L_{r\sigma}) \cdot L_m(\lambda_{dq}) + L_{s\sigma} L_{r\sigma}],$$

$$\omega_5 = R_r \cdot [L_m(\lambda_{dq}) + L_{s\sigma}] / [(L_{s\sigma} + L_{r\sigma}) \cdot L_m(\lambda_{dq}) + L_{s\sigma} L_{r\sigma}],$$

$$\omega_2 = \omega_{\text{ref}} \,, \quad \omega_6 = \omega_{\text{ref}} - \omega_r \,.$$

It can now be observed that in conjunction with dependency (21), structure (22), with *d-q* axis voltages as inputs, represents a system of ordinary differential equations. Thus, the structure (22) is prepared for integration into power systems analyzers.

VI. Numerical Example

For a 3.5 kW induction machine, in Fig. 1 are depicted the magnetizing curve and magnetizing inductance, both given as function of magnetizing current (9). The curves correspond to parameters α = 2.8, β = 5.7 of representation (10). The value of parameter (11) is here L_p = 4 mH. However, merely with the values of α and β, (21) is established, being depicted in Fig. 2.

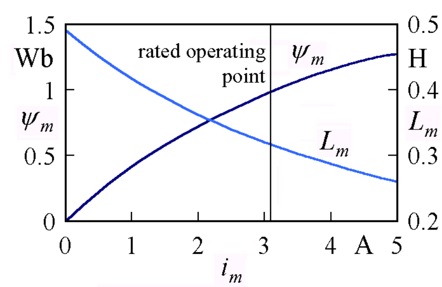

Fig. 1. Machine magnetizing curve together with magnetizing inductance, given as function of magnetizing current (9).

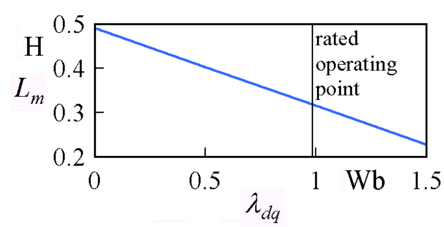

Fig. 2. Magnetizing inductance as function of flux-dependent quantity (18).

References

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