Analysis of a Self-Excited Induction Generator with P-Q Load Model

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***Abstract*—A simple method of analyzing the steady state performance of a self-excited induction generator with *P-Q* load model is described. First the basic operating equations of the generator are derived from its equivalent circuit and then solved using a numerical based routine. The proposed method is then tested on a 1.5-kW induction generator and the simulation results found are compared with the corresponding experimental values.**

***Index Terms*—Induction generators, *P-Q* load model.**

I. Introduction

SELF-excited induction generators (SEIG) are increasingly being used in small power plants in remote areas because of their many advantageous features over synchronous generators. A SEIG obtained its excitation reactive power from a local capacitor bank. Unlike a grid connected induction generator, the voltage and frequency of a SEIG are not constant and that make the analysis of a SEIG much more difficult than that of a grid connected generator.

The steady state performance of a SEIG is usually determined from its equivalent circuit. In almost all of the previous studies, the load is represented by an impedance [1]-[6]. The equivalent circuit of the generator, including the load impedance, is then analyzed either by the loop impedance approach [2]-[5] or the nodal admittance approach [2], [6]. In both approaches, two nonlinear equations are first derived and then solved for two unknowns. In most cases, the equations are explicitly expressed in terms of actual unknowns and that requires rigorous algebraic manipulations. However, the additional algebraic manipulations can be avoided by employing a special routine in solving the equations [3].

The impedance load model is highly questionable. In power system analysis, a load is usually represented by a sink of active power (P) and reactive power (Q), and is considered to be more realistic. When such a *P-Q* load model is used, the conventional loop impedance or nodal admittance approach may not be used to analyze the problem. Thus, it is necessary to reformulate the problem with *P-Q* load model.

This letter describes a simple method of formulating the problem of a SEIG with *P-Q* load model. The problem is then solved using the 'fsolve' routine given in MATLAB.

II. Problem Formulation

The equivalent circuit of a SEIG (with usual notations) including the excitation capacitor reactance X_c and the load (P+jQ) is shown in Fig. 1. F and ω represent the per unit (pu) frequency and pu speed, respectively, of the generator. Note that the magnetizing reactance X_m of the generator is not constant but depends on the ratio of air-gap voltage (V_g) to pu frequency (F). In this study, V_g/F is considered as [3]

$$V_g / F = k_0 + k_1 X_m + k_2 X_m^2 + k_3 + X_m^3 \tag{1}$$

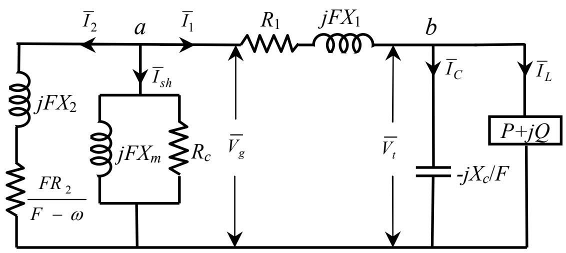

Fig. 1 Per-phase equivalent circuit of a three-phase SEIG.

The KCL at nodes 'a' and 'b' in Fig. 1 can be written as

$$\bar{I}_1 + \bar{I}_2 + \bar{I}_{sh} = 0 \quad \text{and} \quad \bar{I}_1 - \bar{I}_c + \bar{I}_L = 0 \tag{2}$$

The load current $\bar{I}_L$ can be expressed as

$$\bar{I}_L = (P - jQ) / \bar{V}_t^* \tag{3}$$

Here $\bar{V}_t$ is the terminal voltage or load voltage. From (2) and (3), one can easily obtain the following equation

$$(P - jQ) = \bar{V}_t^* (\bar{I}_2 + \bar{I}_{sh} + \bar{I}_c) \tag{4}$$

where $\bar{I}_2 = \dfrac{V_g}{FR_2/(F-\omega) + jFX_2}$; $\quad \bar{I}_{sh} = \dfrac{V_g}{R_c} + \dfrac{V_g}{jFX_m}$

$$\bar{V}_t = V_g + (\bar{I}_2 + \bar{I}_{sh})(R_1 + jFX_1)\,; \quad \bar{I}_c = \bar{V}_t / (-jX_c/F)$$

Here V_g is considered as reference. By separating the real and imaginary parts of (4), the following equations can be obtained

$$g_1 = P - real\left[\bar{V}_t^* \left(\bar{I}_2 + \bar{I}_{sh} + \bar{I}_c\right)\right] = 0 \tag{5}$$

$$g_2 = Q + imag\left[\bar{V}_t^* \left(\bar{I}_2 + \bar{I}_{sh} + \bar{I}_c\right)\right] = 0 \tag{6}$$

Equations (5) and (6) represent the basic equations and must be satisfied for all operating conditions of the generator.

When the load is composed of constant impedance (Z), constant current (I) and constant power (P) components, the active and reactive power can be expressed as [7]

$$P = P_0 \left[a_1 (V_t / V_0)^2 + a_2 (V_t / V_0) + a_3\right] \tag{7}$$

$$Q = Q_0 \left[b_1 (V_t / V_0)^2 + b_2 (V_t / V_0) + b_3\right] \tag{8}$$

Here subscript 'o' in P, Q and V is used to represent the values at initial operating condition. The above load model is also called the *ZIP* load model. In order to incorporate the *ZIP* load model in the proposed method, P and Q of (5) and (6) are to be substituted from (7) and (8), respectively.

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III. Solution Technique

Solution of eqns. (5) and (6), in conjunction with (1), can provide the values of only two unknowns. However, the circuit of Fig. 1 has five unknowns: X_m, F, ω, X_c, and P (for a given power factor). Thus, it is necessary to assign some feasible values to three of the unknowns. In general, the equations are solved to find the values of X_m and F for given values of X_c, ω and load. Under such a case, eqns. (5) and (6) can be rewritten in the following general form

$$\mathbf{G}(\mathbf{X}) = 0 \tag{9}$$

Here $\mathbf{G} = [g_1 \;\; g_2]^T$ and $\mathbf{X} = [X_m \;\; F]^T$. Knowing the values of X_m and F, the voltage, current and power at various points of the circuit can easily be determined.

Expressing (5) and (6) in terms of actual unknowns X_m and F requires rigorous algebraic manipulations. However, when a numerical based routine is used to solve the equations, no such manipulations are needed. In this study, (9) is solved using the 'fsolve' routine given in MATLAB and it uses a nonlinear least-squares algorithm. One of the characteristics of the least-squares based algorithm is that when the system of equations does not have a numerical zero point, the routine may still return to a point where the residual is small. The residual (σ) of (9) may be defined as

$$\sigma = \sqrt{\left(g_1^2 + g_2^2\right)/2} \tag{10}$$

When the basic equations (5) and (6) of the generator are satisfied, the residual σ should have a zero value for an ideal case or less than a very small threshold value ε (say, 10^{-6}) for a practical case. When $\sigma > \varepsilon$, the corresponding solution may not be considered as acceptable and such a situation may arise when the set of assigned values of the unknowns or adjustable parameters is not feasible.

IV. Results and Discussions

The proposed method of analyzing the performance of a SEIG with *P-Q* load model is tested on a three-phase, 1.5-kW induction generator. The parameters of the generator are given in [3]. First eqn. (9) is repeatedly solved for various values of P (from zero to 1.5 kW with an increment of 5 W) at unity power factor with C = 37 μF and ω = 1.0 pu. The distribution of residuals found is shown in Fig. 2 and it indicates that, for $P \leq$ 865 W, the residual is insignificant ($< 10^{-10}$) indicating that the routine successfully converged to the numerical zero point. However, for $P \geq$ 870 W, the residual is much higher (0.0076 ~ 74.34) indicating that there is no numerical zero of (9). In other words, the generator is unable to supply such a high power with the assigned values of parameters. The variation of load voltage against power is shown in Fig. 3 and it indicates that the voltage decreases with load, as expected. Fig. 3 also indicates that the experimental values are very close to the corresponding simulation results. During experiment, the maximum power is found as 873 W and is close to the corresponding simulation result of 865W.

When the *ZIP* load model (with $a_1 = a_2 = 0.15$ and $a_3 = 0.70$) at unity power factor is used, the method successfully converged to the zero point for $P_0 \leq$ 940 W. For this load model, it is difficult to verify the results experimentally.

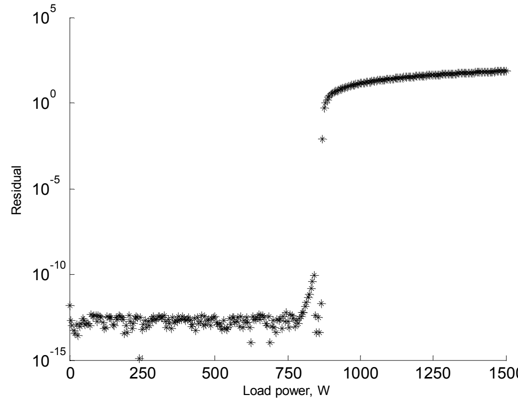

Fig. 2 Distribution of residuals.

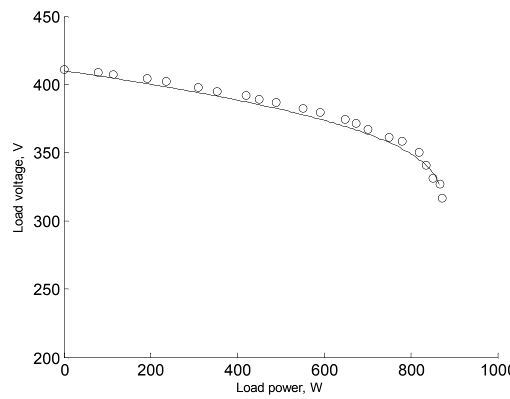

Fig. 3 Variation of load voltage against load power for the converged cases. '–' simulation results; 'o' experimental results

V. Conclusions

A simple method of evaluating the steady state characteristics of a three-phase self-excited induction generator with *P-Q* load model is systematically described in this letter. First the problem of the generator is formulated through its equivalent circuit using the basic circuit laws. The formulated problem is then solved using a numerical based routine to avoid the additional algebraic manipulations of the equations. The proposed method is then tested on a 1.5-kW machine and the simulation results are found to be in excellent agreement with the corresponding experimental values.

VI. References

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