Homoclinic Loop and Degenerate Hopf Bifurcations Introduced by Load Dynamics

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***Abstract*—This letter presents the occurrence and properties of a homoclinic loop bifurcation in a simple power system with Induction Motor and LTC. This global bifurcation is accompanied by the emergence of an unstable limit cycle, or the disappearance of a stable one. It is also shown that it may coincide with a Hopf bifurcation condition yielding a codimension-2 degenerate Hopf bifurcation.**

***Index Terms*—bifurcation surface, degenerate Hopf bifurcation, homoclinic loop, load dynamics.**

I. Introduction

GLOBAL bifurcations have been observed in early papers even for large scale power systems [1]. In this paper first order models of an Induction Motor (IM) and a Load Tap Changer (LTC), assumed to be continuous, [2] are used to study load dynamics that for certain loading values lead to a Homoclinic Loop Bifurcation (HLB). A similar system was used for the study of local bifurcations in [3].

The HLB occurs when one branch of the unstable manifold of an unstable (saddle) equilibrium point (uep) coincides with one branch of the stable manifold of the same equilibrium. Thus, an isolated closed trajectory is formed which starts from the uep for $t \to -\infty$ and tends it for $t \to \infty$. This is a global bifurcation, as the general structure of the system changes at the bifurcation [4].

As in the Hopf Bifurcation (HB) case, the HLB can be sub- or super-critical. In the supercritical HLB case, an unstable limit cycle is generated after the bifurcation. In the subcritical HLB case, a stable limit cycle (generated, for instance, through a supercritical HB) vanishes at the bifurcation.

The HLB is a codimension-1 bifurcation, thus it requires one single condition. If the HLB condition coincides with an HB condition, the resulting bifurcation is a codimension-2 Degenerate Hopf Bifurcation (DHB) [4].

II. Bifurcation Surface

The test system configuration is shown in Fig. 1 with data given in Table I. It incorporates an infinite bus (constant voltage source) which feeds a composite load consisting of an IM in parallel with an admittance load through a lossless line and a transformer equipped with an LTC. The first order dynamic models of IM and LTC are taken from [2], with state variables motor slip s and LTC tap ratio r.

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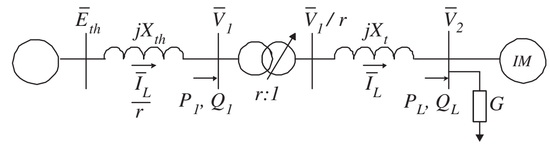

Fig. 1. Test power system

TABLE I
System Parameters (pu, sec)

R_s	X_s	R_r	X_r	X_m	E_{th}	X_{th}	X_t	H	T_L
0.062	0.2	0.036	0.36	6.4	1.4	1.2	0.15	0.5	100

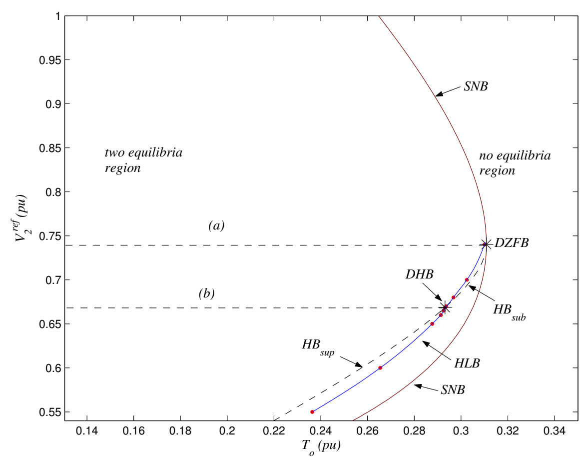

Fig. 2. Parameter space exhibiting bifurcation emergence

The bifurcation surface for the system under study is shown in the parameter space (T_o, V_2^{ref}) in Fig. 2, considering a loading pattern of a simultaneous increase of IM torque at nominal speed T_o and load conductance G. Note that mechanical torque is considered quadratic. SNB, HB, and HLB curves denote saddle-node, Hopf and homoclinic loop bifurcation conditions respectively. The HLB branch is computed point by point using trial and error. Point *DZFB*, where SNB and HB (as well as HLB) branches intersect corresponds to a double-zero fold bifurcation [5], while point *DHB* is a degenerate Hopf. The area to the left of the SNB curve corresponds to the feasible region of the system, where two equilibria exist. Below DZFB, the stability boundary is formed by the HB branch, i.e. the operating points between HB and SNB branches are unstable.

III. Phase Portraits

In Fig. 3 the phase portrait at a supercritical HLB for $V_2^{ref} = 0.74$ pu is shown. This voltage level is between lines

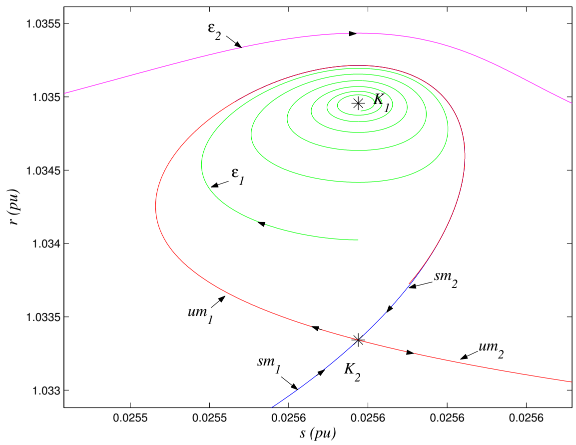

Fig. 3. System trajectories at homoclinic loop bifurcation

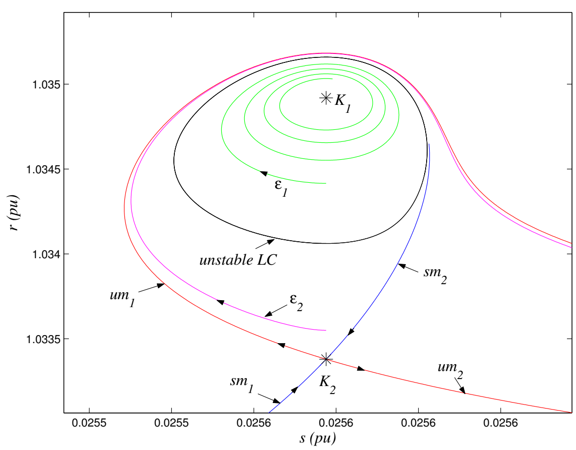

Fig. 4. Unstable limit cycle before subcritical HB ($V_2^{ref} = 0.74$ pu)

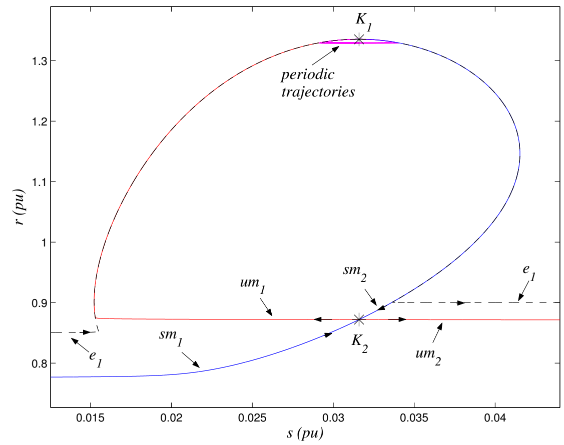

Fig. 5. Phase portrait for the case of degenerate Hopf bifurcation

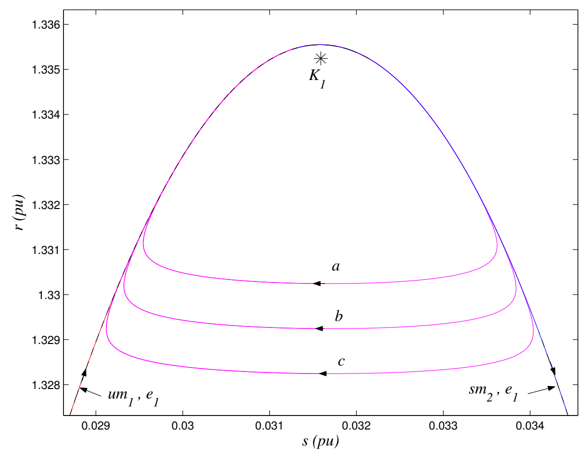

Fig. 6. Non-isolated periodic trajectories encircling equilibrium K_1 (center)

(a) and (b) of Fig. 2, and therefore the HLB occurs prior to the HB (which is subcritical). The homoclinic loop is formed by one branch of the unstable manifold (um_1) and one branch of the stable manifold (sm_2) of uep K_2, and it bounds the region of attraction of the stable equilibrium K_1. Note that load dynamics around K_1 are already oscillatory at the HLB.

The birth of the unstable limit cycle is accompanied by the breaking of the homoclinic loop and the creation of an escape path through which trajectories drift away. This is shown in Fig. 4 for slightly increased loading conditions after the HLB, but before the subcritical HB. Now the region of attraction of K_1 is determined by the shrinking unstable limit cycle.

As seen from Fig. 2, branches HLB and HB intersect at the Degenerate Hopf Bifurcation point *DHB*. Above point *DHB* the HB is subcritical, while below this point it is supercritical. In the latter case the stable limit cycle created after the HB grows for increased load and eventually disappears at the subcritical HLB (see Fig. 2).

In Fig. 5 the phase portrait at the DHB is shown. Inside the homoclinic loop (branches um_1 and sm_2 of uep K_2) all trajectories are periodic orbits encircling the stable equilibrium K_1, which becomes a center. Trajectory e_1 (drawn with dashed line) is just outside the homoclinic loop and diverges. Three periodic trajectories *a-c* are shown in detail in Fig. 6.

IV. Conclusion

This letter shows that the oscillatory behavior of load dynamics can lead to the occurrence of a global Homoclinic Loop Bifurcation. It was seen that the HLB exists either before a subcritical HB, or after a supercritical HB and is responsible for the emergence or disappearance of the limit cycle interacting with the HB. When HLB and HB conditions appear simultaneously, they give rise to a degenerate HB also presented in this letter.

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