A New Algorithm to Photovoltaic Power Point Tracking Problems with Quadratic Maximization

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Abstract- One of critical concerns in the operation of the photovoltaic system is the maximum power point (MPP) tracking problem that increases the economical feasibility of the system. This letter presents a tracking method using the quadratic polynomial to perform a MPP tracker of a given PV array. With the method, the maximum power calculation is made from a quadratic manner that it convergences significantly accelerated.
Keywords: maximum power point, quadratic polynomial

I. Introduction

According to the intrinsic characteristics of solar cell, there exists an optimal operating condition for the photovoltaic (PV) system to be followed in order to extract maximum power from it, and this leads to the development of maximum power point tracking (MPPT) algorithm. There are several MPPT methods existing in the literatures [1-4]. In those methods, perturbation and observation (P&O) method is a well-known iterative method. With periodically changing the PV output voltage, one can observe the MPP operating condition. The P&O method is often used for its simplicity. However, in a fast changing irradiance condition, the PV system responses the change of environment as the effect of perturbation of the operating voltage, and it sometimes leads to a wrong tracking direction to the MPP. Under the condition, Petrone *et al.* detail the perturbing step and sampling period of data acquisition for the P&O method to reliably track the variable irradiance [1]. Via the optimization of the perturbation amplitude with parabolic prediction, an adaptive step-size tracking scheme is also recommended and shows better performance for the increase of the power drawn from PV field.

In this letter, a MPPT method based on the second-order Lagrange interpolating polynomial is suggested. This method utilizes the MPP calculation from a quadratic form that three known duty cycles are first collected. Then, the convergent conditions can be proven via an additional sorting loop to take the current largest power output being at the second known point. This follows by using a local maximum value of the approximate quadratic function to release the smallest known power point, the proposed iterative can be further accelerated. To validate the effectiveness of the method, several cases are examined. Test results indicate that the proposed method can operate under various conditions with a faster response.

II. MPP Tracking Method Based on Quadratic Polynomial

(*a*) *Basic concepts*

It supposes that $x_o, x_1,..,x_n$ are $(n+1)$ distinct numbers and f is a function whose values are given at these numbers. Then, there exists a Lagrange interpolating polynomial $P_n(x)$ of degree at most n satisfying $f(x_k)=P_n(x_k)$ for each $k=0, 1,\dots, n$, and $P_n(x)$ is given as

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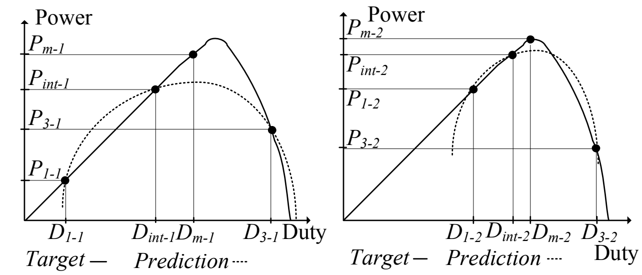

(a) first iteration (b) second iteration

Fig. 1: Iteration algorithm using a quadratic polynomial to find the maximum power point of a PV system

$$P_n(x) = \sum_{k=0}^{n} f(x_k) L_{n,k}(x) \qquad (1)$$

where

$$L_{n,k}(x) = \prod_{\substack{i=0 \\ i\neq k}}^{n} \frac{(x-x_i)}{(x_k-x_i)} \qquad (2)$$

In eq. (1), $P_n(x)$ can be simplified as a quadratic function by setting n=2. In this case, to find the extreme value of such a quadratic polynomial $P_2(x)$, it requires

$$\left.\frac{dP_2}{dx}\right|_{x=x_{\max}} = 0 \qquad (3)$$

Thus

$$x_{\max} = \frac{1}{2}\frac{x_0^2[f(x_2)-f(x_1)]+x_1^2[f(x_0)-f(x_2)]-x_2^2[f(x_1)-f(x_0)]}{x_0[f(x_2)-f(x_1)]+x_1[f(x_0)-f(x_2)]+x_2[f(x_1)-f(x_0)]} \qquad (4)$$

If three given data are located in the neighborhood of the extreme value, by using equations, the quadric polynomial offers a simple way to represent the value of $P_2(x)$ near its maximum point. This gives the idea for a better photovoltaic MPPT method via three measurement results.

(*b*) *Applying quadratic polynomial to the photovoltaic MPPT*

Let the solid curve in Fig 1(a) be the characteristic curve of a PV array, and three working conditions with measured power information be (D_{1-1}, P_{1-1}), (D_{int-1}, P_{int-1}) and (D_{3-1}, P_{3-1}), where the second subscript of variable name represents the iteration step. Using the above information, a quadratic polynomial simulating the power output can be found by using eq. (1), and the duty cycle, D_{m-1} giving its predicted maximum power output is calculated by eq. (4). But, the actual power output from the PV array is P_{m-1} at D_{m-1}. At this step, four different power points P_{1-1}, P_{int-1}, P_{3-1} and P_{m-1} have been obtained.

Next, by dropping out the lowest power point of the four, the second iteration can be launched with refreshed three points (D_{1-2}, P_{1-2}), (D_{int-2}, P_{int-2}) and (D_{3-2}, P_{3-2}). As shown in Fig. 1(b), the quadratic polynomial at the second iteration step using the latest three-point set, and the power P_{m-2} at D_{m-2} is obtained similarly to the previous step. This iterative scheme produces a sequence of P_{m-1}, P_{m-2} ,…, P_{m-n} and it stops until the difference of two successive iteration results is less than a predefined value which determines the convergence of the method.

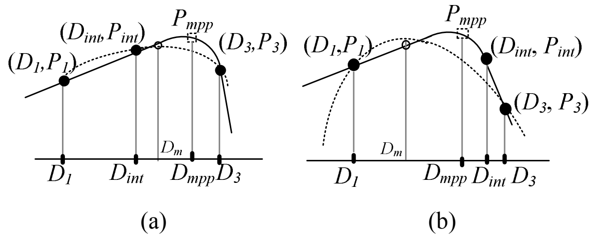

Fig. 2: Conceptual sketch for the predicted curve

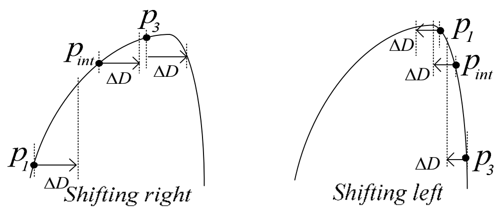

Fig.3: Duty shifting for monotonically Inc. /Dec. power output

(*c*) *Fast convergent conditions*

As seen from Fig. 2, the actual PV curve can be viewed that if D is less than D_{mpp}, the duty condition giving maximum power output, the curve is almost a linearly monotonic increasing function. Also if the duty cycle D goes over D_{mpp}, the curve becomes a monotonic decrease. Based on the fact, firstly let the observed three duties satisfy the condition, $D_1 < D_{int} < D_3$; and give that the power P_{int} is a current largest power output at three known power. In this case,

(i) If $D_1 < D_{int}$ and $P_{int} > P_1$, the derivative $P'(D)$ on the interval of D_1 and D_{int} is positive.

(ii) If $D_{int} < D_3$ and $P_{int} > P_3$, $P'(D)$ becomes negative on the interval of D_{int} and D_3.

According to the first derivative test of the calculus, it states if $P'(D)$ changes from positive to negative on the interval of D_1 and D_3, then a value D_m depositing between D_1 and D_3 can be found while satisfying the power at D_m is a local maximum. It implies the point (D_m, P_m) is a vertex with down-turned concavity. The corresponding local maximum power P_m can be thus used in place of the lowest power of the known points.

Based on the concepts, Fig. 2(a) and 2(b) demonstrate two desired tracking conditions at which convergent results can be quickly reached. In Fig. 2(a), two of the measured information, (D_1, P_1) and (D_{int}, P_{int}) are located at the up-hill side of the PV characteristic curve and the third point (D_3, P_3) is at the downhill side of the curve. On the other hand, Fig. 2(b), (D_1, P_1) is at the up-hill side of the PV curve and the other two data are at the down-hill side of the curve. The above two examples has one thing in common: D_{mpp} , giving the true largest power of solar array, is always located between D_1 and D_3, and in both cases the lowest power-points, P_1 or P_3 appears at the boundary. In other words, D_1 or D_3 with the smallest power output will be dropped out during the iterative step. It implies the remaining points are much closer to the target D_{mpp} after each iterative loop, and it converges to the theoretical MPP effectively.

However, if the three measured information are all located at either side of the PV curve, up-hill or down-hill, extra considerations shall be enforced in order to have a convergent maximum power tracking result. The idea is to implement a sub-loop to confirm the current largest power output being at D_{int}. Fig. 3 gives the diagrams of possible duty adjustment, performed to ensure the power measurement at D_{int} is always the largest. When P_1 is the largest, it implies that three working duties are in the down-hill side of the PV curve(monotonically decreasing), every working duty needs to be shifted to the left by a given step size to ensure the convergence result. Similarly, all duties are shifted to the right if P_3 is found to be the largest.

Table 1: Simulated tracking results with cRIO instrumentation using pre-calculated PV characteristic curves

Insolation	TMPP Duty (%)	IMPP Duty (%)	Error (%)	Quadratic Loop	Adjust Loop	MPPT Times(ms)
A	64.0	64.1	0.2	4	8	12
B	66.0	67.3	2.0	3	3	5
C	69.0	69.8	1.2	3	4	6
D	71.0	70.7	-0.4	5	24	28

Remarks:
TMPP: Theoretical MPP; IMPP: Iterative MPP; Quadratic Loop: Iterative numbers for the main loop; Adj. Loop: Iterative numbers for duty adjustment; MPPT Times: the recorded running time

IV. Case Studies

To validate the proposed method, a 25W PV panel providing 10-18V output and a buck converter converting PV power to charge the voltage rating 6V battery and to give necessary power for DC motors running as a load requirement, is set up in laboratory. In the control level, the prototype is realized by the hardware from cRIO module, National Instruments(NI). For the software development, the MPP tracking algorithm is coded by LabVIEW. The MPPT controller controls the PWM duty control using the proposed quadratic maximization algorithm in order to make the buck converter operate at the MPP output condition.

In the test, there are previously scanned records for the solar characteristic curves approximated by LabVIEW toolkit to form a set of curve equations which are later used as the theoretical curves of the PV system. With these curves in hand, one can perform the quadratic MPPT simulation with the real time computing system, cRIO for it accuracy test, and realize how fast it can reach to the maximum power point. Table 1 indicates that using various theoretical PV characteristic curves, the proposed MPP tracking needs only less than 30ms to find the optimum duty cycle for maximum power output of the PV system; and the estimated duty has an error less than 2%. Although there is still room for improvement, especially for the entire circuitry arrangement, the current setup is believed enough for practical usage.

V. Conclusions

In this letter, we have proposed a method using quadratic estimation MPPT algorithm to maximize the power transforming efficiency of a given PV array. At this time, this prototype implemented from the proposed approach is being assembled for a leisure-typed electrical boat with auxiliary solar energy. Test results will be reported in the near future.

VI. References

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