

Optimal and Autonomous Incentive-based Energy Consumption Scheduling Algorithm for Smart Grid

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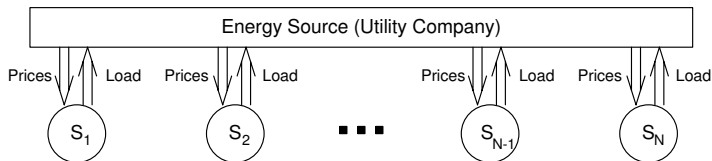
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OUTLINE

- Traditional Load Control vs. Load Control in Smart Grid
- System Model and Problem Formulation
- Energy Consumption Game
- Simulation Results
- Conclusions

TRADITIONAL LOAD CONTROL



- Some variations:

- ▶ Direct load control
- ▶ Real-time Pricing
- ▶ Conservation Rates
- ▶ Etc.

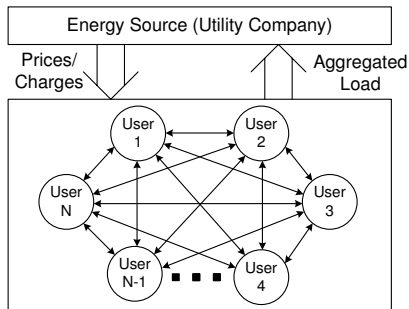
- Key feature:

- ▶ The focus of load control is on the interactions between the utility and **each** user.

WHAT DOES THE UTILITY CARE ABOUT?

- The utility cares about the **aggregated** behavior of all users:
 - ▶ The total load at each hour.
 - ▶ The peak-to-average ratio (PAR) in total load demand.
- Therefore, the current setting does not seem to be the best choice.

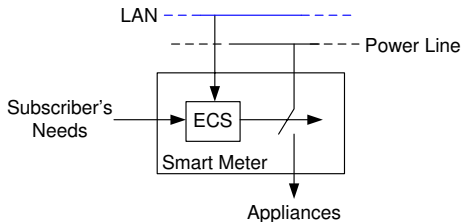
LOAD CONTROL IN SMART GRID



- In this setting, the users also interact with **each other** through **message exchanges**.
- Message exchanges can be:
 - ▶ Manual \Rightarrow Phone, E-mail, etc.
 - ▶ Automatic over a **two-way communication infrastructure** \Rightarrow “Smart Grid”.

OUR FOCUS

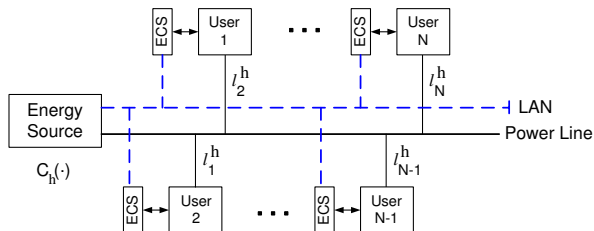
- To design an autonomous and incentive-based load control system:
 - ▶ We deploy **energy consumption scheduling** (ECS) units in “Smart Meters”.
 - ▶ Users should have **incentives** to use ECS units \Rightarrow “Smart Pricing”.



- ECS unit schedules energy consumption of all/most household appliances.

SYSTEM MODEL: SINGLE SHARED ENERGY SOURCE

- We consider a scenario where a source of energy is **shared** by several users, each one equipped with an ECS-enabled smart meter.



- Here, we have:
 - ▶ Energy source: **generator**, step-down **substation** transformer, etc.
 - ▶ l_n^h : Power load of user $n \in \mathcal{N} = \{1, \dots, N\}$ at hour $h \in \mathcal{H} = \{1, \dots, 24\}$.
 - ▶ $C_h(\sum_{n \in \mathcal{N}} l_n^h)$: Energy cost at hour $h \in \mathcal{H}$ as a function of **total hourly** load.

SYSTEM MODEL: HOUSEHOLD APPLIANCES

- For each user $n \in \mathcal{N}$, the set of appliances is shown by \mathcal{A}_n .
- For each $a \in \mathcal{A}_n$, we define
 - ▶ $E_{n,a}$: pre-determined total daily energy consumption required.
- Example: $E_{n,a} = 9$ kWh for a PHEV for a 40-mile daily driving range.
- Clearly, we have

$$\sum_{h \in \mathcal{H}} l_n^h = \sum_{a \in \mathcal{A}_n} E_{n,a}, \quad \forall n \in \mathcal{N}.$$

- We want to **shift** consumption to reduce PAR, **not** to **reduce** consumption!

SYSTEM MODEL: SCHEDULING

- For each user $n \in \mathcal{N}$ and each appliance $a \in \mathcal{A}_n$, we define

$$\mathbf{x}_{n,a} \triangleq [x_{n,a}^1, \dots, x_{n,a}^{24}],$$

where $x_{n,a}^h$ denotes the scheduled **one-hour** energy consumption for appliance a .

- The ECS unit's job is to choose the right values for $\mathbf{x}_{n,a}$ for each $a \in \mathcal{A}_n$.
 - ▶ It should satisfy the users' needs.

SYSTEM MODEL: USER'S NEEDS

- To address user n 's **needs**, we further define
 - ▶ $\alpha_{n,a}$: Beginning of the time interval that consumption can be scheduled.
 - ▶ $\beta_{n,a}$: End of the time interval that consumption can be scheduled.
 - ▶ $\gamma_{n,a}^{\min}, \gamma_{n,a}^{\max}$: Minimum and maximum scheduled power levels.
- For each user $n \in \mathcal{N}$, we can define **feasible scheduling set** as follows:

$$\mathcal{X}_n = \left\{ \mathbf{x}_n \mid \sum_{h=\alpha_{n,a}}^{\beta_{n,a}} x_{n,a}^h = E_{n,a}, \quad \forall a \in \mathcal{A}_n, \right. \\ \left. x_{n,a}^h = 0, \quad \forall h \in \{1, \dots, \alpha_{n,a} - 1\} \cup \{\beta_{n,a} + 1, \dots, 24\}. \right. \\ \left. \gamma_{n,a}^{\min} \leq x_{n,a}^h \leq \gamma_{n,a}^{\max}, \quad \forall h \in \{\alpha_{n,a}, \dots, \beta_{n,a}\} \right\}.$$

where $\mathbf{x}_n \triangleq [x_{n,a}, \forall a \in \mathcal{A}_n]$. The ECS unit may **only** select $\mathbf{x}_n \in \mathcal{X}_n$.

SYSTEM MODEL: ENERGY COST

Question: Which is the best feasible choice of \mathbf{x}_n for each user $n \in \mathcal{N}$?

- Consider the **total** load in the system at **each hour** of the day $h \in \mathcal{H}$:

$$L_h \triangleq \sum_{n \in \mathcal{N}} l_n^h = \sum_{n \in \mathcal{N}} \sum_{a \in \mathcal{A}_n} x_{n,a}^h,$$

- We define an **energy cost function** $C_h(L_h)$ indicating the cost of generating or providing energy by the energy source at each hour $h \in \mathcal{H}$.
- In general, the energy cost function can be different at different hours of the day.
- For example, the cost can be less at **night** compared to the **day** time.
- The cost can be **actual** or **artificial**, i.e., for load shaping.

SYSTEM MODEL: ENERGY COST

- We assume that the energy cost functions are **increasing** and **convex**

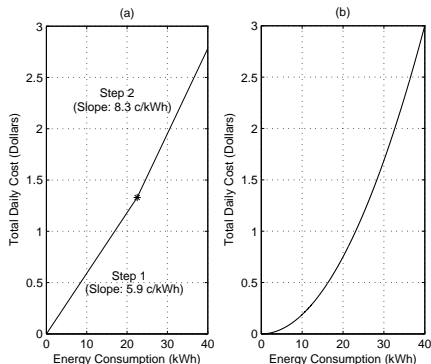


Figure: Two sample convex and increasing cost functions: (a) Two-step conservation rate model used by BC Hydro; (b) A quadratic cost function forming a similar shape.

OPTIMIZATION PROBLEM

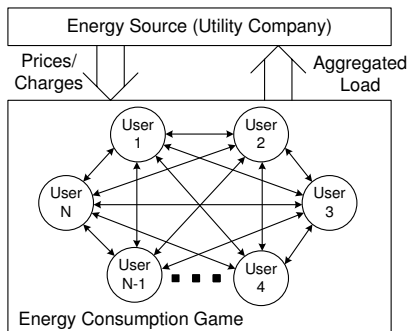
- Given **complete knowledge** about the users' needs and a **centralized control** of the system, an efficient energy consumption scheduling to be implemented by the ECS units can be characterized as the solution of the following problem:

$$\begin{aligned} & \underset{\mathbf{x}_1, \dots, \mathbf{x}_N}{\text{minimize}} && \sum_{h=1}^{24} C_h \left(\sum_{n \in \mathcal{N}} \sum_{a \in \mathcal{A}_n} x_{n,a}^h \right) \\ & \text{subject to} && \mathbf{x}_n \in \mathcal{X}_n, \quad \forall n \in \mathcal{N}. \end{aligned} \tag{1}$$

- Problem (1) has several **constraints** which are encapsulated in sets \mathcal{X}_n .
- Problem (1) is a **convex** optimization problem.
- Problem (1) has a **unique** optimal solution if the cost function is **strictly** convex.

DESIGN OBJECTIVE

- Solve Problem (1) in a **distributed** fashion with **cooperation** from the users.
 - ▶ A good pricing model can encourage users to cooperate for their own benefits.



- The system can be modeled as an **energy consumption game**.

PRICING AND BILLING MODEL

- Let b_n denote what appears on the **bill** for user $n \in \mathcal{N}$ at the **end of the day**.
- In general, we would expect two properties to hold in the pricing model.
- **Property I** - The total daily charge should be as high as the total daily cost:

$$\sum_{n \in \mathcal{N}} b_n \geq \sum_{h=1}^{24} C_h \left(\sum_{n \in \mathcal{N}} l_n^h \right),$$

That is

$$\kappa \triangleq \frac{\sum_{n \in \mathcal{N}} b_n}{\sum_{h=1}^{24} C_h \left(\sum_{n \in \mathcal{N}} l_n^h \right)} \geq 1.$$

- If $\kappa = 1$, then the system is **budget balanced**. But usually, $\kappa > 1$.

PRICING AND BILLING MODEL

- **Property II** - We have:

$$\frac{b_n}{b_m} = \frac{\sum_{h=1}^{24} l_n^h}{\sum_{h=1}^{24} l_m^h} = \frac{\sum_{a \in \mathcal{A}_n} E_{n,a}}{\sum_{a \in \mathcal{A}_m} E_{m,a}}, \quad \forall n, m \in \mathcal{N}.$$

- After reordering the terms, we have

$$\sum_{m \in \mathcal{N}} b_m = \sum_{m \in \mathcal{N}} \left(b_n \frac{\sum_{h=1}^{24} l_m^h}{\sum_{h=1}^{24} l_n^h} \right) = b_n \frac{\sum_{m \in \mathcal{N}} \sum_{h=1}^{24} l_m^h}{\sum_{h=1}^{24} l_n^h}.$$

PRICING AND BILLING MODEL

- There is only a **single pricing model** which satisfies Properties I and II:

$$\begin{aligned} b_n &= \frac{\sum_{h=1}^{24} l_n^h}{\sum_{m \in \mathcal{N}} \sum_{h=1}^{24} l_m^h} \left(\sum_{m \in \mathcal{N}} b_m \right) \\ &= \frac{\kappa \sum_{h=1}^{24} l_n^h}{\sum_{m \in \mathcal{N}} \sum_{h=1}^{24} l_m^h} \left(\sum_{h=1}^{24} C_h \left(\sum_{m \in \mathcal{N}} l_m^h \right) \right) \\ &= \frac{\kappa \sum_{a \in \mathcal{A}_n} E_{n,a}}{\sum_{m \in \mathcal{N}} \sum_{a \in \mathcal{A}_m} E_{m,a}} \left(\sum_{h=1}^{24} C_h \left(\sum_{m \in \mathcal{N}} \sum_{a \in \mathcal{A}_m} x_{m,a}^h \right) \right). \end{aligned}$$

- The charge to one user also depends on **other** users schedules!
- That explains why it is a game among users.

ENERGY CONSUMPTION GAME

We can formally define the energy consumption game among the users as:

- **Players:** Registered users in set \mathcal{N} .
- **Strategies:** Energy consumption scheduling vectors \mathbf{x}_n for all users.
- **Payoffs:** $P_n(\mathbf{x}_n; \mathbf{x}_{-n})$ for each user $n \in \mathcal{N}$, where

$$\begin{aligned} P_n(\mathbf{x}_n; \mathbf{x}_{-n}) &= -b_n \\ &= -\frac{\kappa \sum_{a \in \mathcal{A}_n} E_{n,a}}{\sum_{m \in \mathcal{N}} \sum_{a \in \mathcal{A}_m} E_{m,a}} \left(\sum_{h=1}^{24} C_h \left(\sum_{m \in \mathcal{N}} \sum_{a \in \mathcal{A}_m} x_{m,a}^h \right) \right). \end{aligned}$$

Here, \mathbf{x}_{-n} denotes the vector of scheduling variables for all users **other than** user n .

BEST RESPONSE

- Given \mathbf{x}_{-n} , each user $n \in \mathcal{N}$ tries to maximize its payoff:

$$\mathbf{maximize}_{\mathbf{x}_n \in \mathcal{X}_n} P_n(\mathbf{x}_n; \mathbf{x}_{-n}). \quad (2)$$

- This is equivalent to the following:

$$\mathbf{minimize}_{\mathbf{x}_n \in \mathcal{X}_n} \sum_{h=1}^{24} C_h \left(\sum_{a \in \mathcal{A}_n} x_{n,a}^h + \sum_{m \in \mathcal{N} \setminus \{n\}} \sum_{a \in \mathcal{A}_m} x_{m,a}^h \right).$$

- User/player n can only change the blue part.
- Playing according to (2) is called **best response** play.

NASH EQUILIBRIUM

- Nash equilibrium is a **solution concept** in game theory.
- The energy consumption scheduling vectors $(\mathbf{x}_n^*, \forall n \in \mathcal{N})$ form a Nash equilibrium if and only if for **each** user/player $n \in \mathcal{N}$,

$$P_n(\mathbf{x}_n^*; \mathbf{x}_{-n}^*) \geq P_n(\mathbf{x}_n; \mathbf{x}_{-n}^*), \quad \forall \mathbf{x}_n \geq 0.$$

- No user/player would try to deviate from a Nash equilibrium.
- Questions to be answered:
 - ▶ Does Nash equilibrium always exist? \Rightarrow **Yes!**
 - ▶ Is it unique? \Rightarrow **Yes!**
 - ▶ Do we converge from any initial point to Nash equilibrium? \Rightarrow **Yes!**
 - ▶ What is the performance at Nash equilibrium? \Rightarrow **Optimal!**

NASH EQUILIBRIUM

- Therefore, to solve problem (1) in a **distributed** fashion:
 - ▶ It is enough to use the **pricing** model explained earlier.
 - ▶ The users will **automatically** find the optimal solution in a **game setting**.

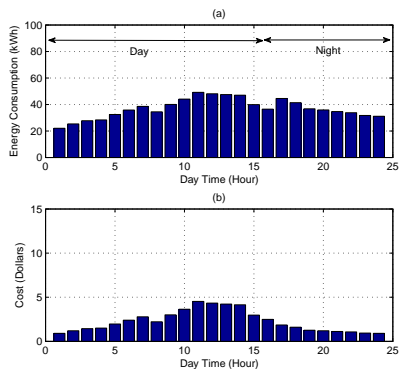
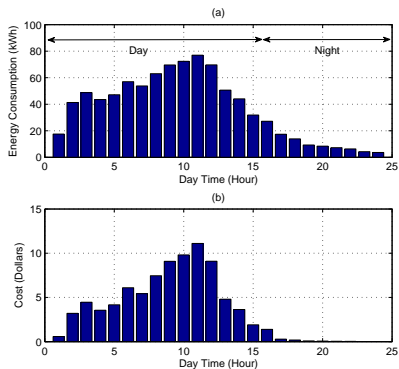
- Another key difference with traditional load control:
 - ▶ Here, the utility does **not** announce the exact price.
 - ▶ Instead, it announces the function that determines the price.

SIMULATION RESULTS: SETTING

Consider the following simulation setting:

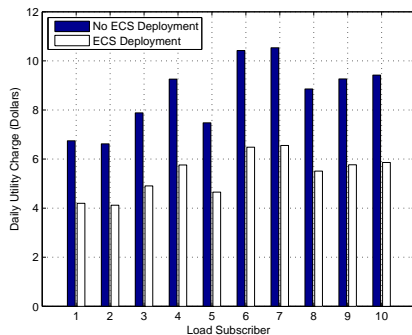
- The number of users is 10.
- Appliances with **hard** energy consumption scheduling constraints:
 - ▶ Refrigerator-freezer (daily usage: 1.32 kWh), electric stove (daily usage: 1.89 kWh for self-cleaning and 2.01 kWh for regular), lighting (daily usage for 10 standard bulbs: 1.00 kWh), heating (daily usage: 7.1 kWh), etc.
- Appliances with **soft** energy consumption scheduling constraints:
 - ▶ Dishwasher (daily usage: 1.44 kWh), clothes washer (daily usage: 1.49 kWh for energy-star 1.94 kWh for regular), clothes dryer (daily usage: 2.50 kWh), and PHEV (daily usage: 9.9 kWh), etc.
- The ECS units schedule consumption **only** for appliances with **soft** constraints.
- Cost function is quadratic with **half price** at night.

SIMULATION RESULTS: COST AND PAR



- **Left:** Results without using ECS units, PAR = 2.1, Cost = \$86.47.
- **Right:** Results when we use ECS units, PAR = 1.3, Cost = \$53.81.

SIMULATION RESULTS: INDIVIDUAL CHARGES



- Every user will benefit from using ECS units.
- Users will have the incentives to participate in ECS program.

CONCLUSIONS

- In Smart Grid, load control is an energy consumption game among users.
- The utility may only announce price functions not exact price values.
- Distributed best response plays will automatically lead to optimal performance.
- The right choice of the pricing model/function is the key.
- Simulation results show reductions in energy cost and peak-to-average-ratio.